Wollo University
Kombolcha Institute of Technology
School of Mechanical & Chemical Engineering

Chapter 3: Mixed Jet Flow

Prepared by: Wubishet Degife.
E-mail: wubesvictory@gmail.com

Kutta-Joukowski Lift Theorem and Conditions

Kutta (German), Joukowski (Russia)

- It is a fundamental theorem in aerodynamics, that can be used for the calculation of the lift of an airfoil on the 2D body including circular cylinder.
- It state that, " *lift per unit depth or span of any cylinder or airfoil immersed in uniform flow stream is equal to* $\rho U_\infty \Gamma$ "

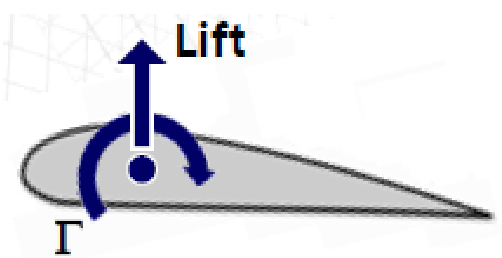
i.e. $L = \rho U_\infty \Gamma$

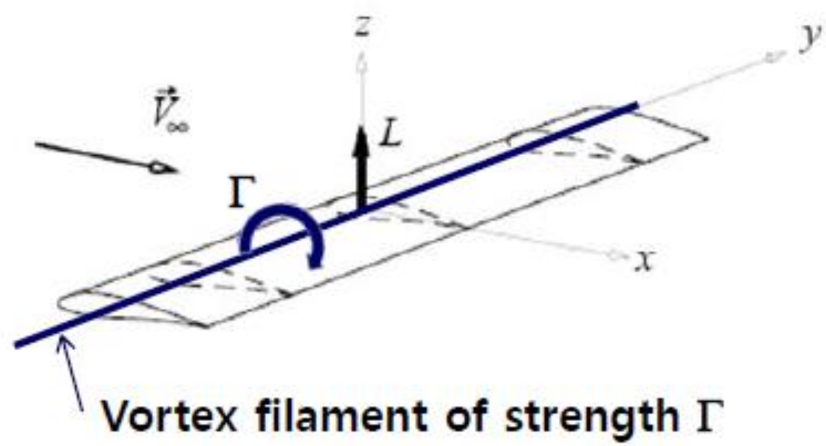
$\Gamma =$ positive clockwise

: ***total net circulation contained within the body shape***

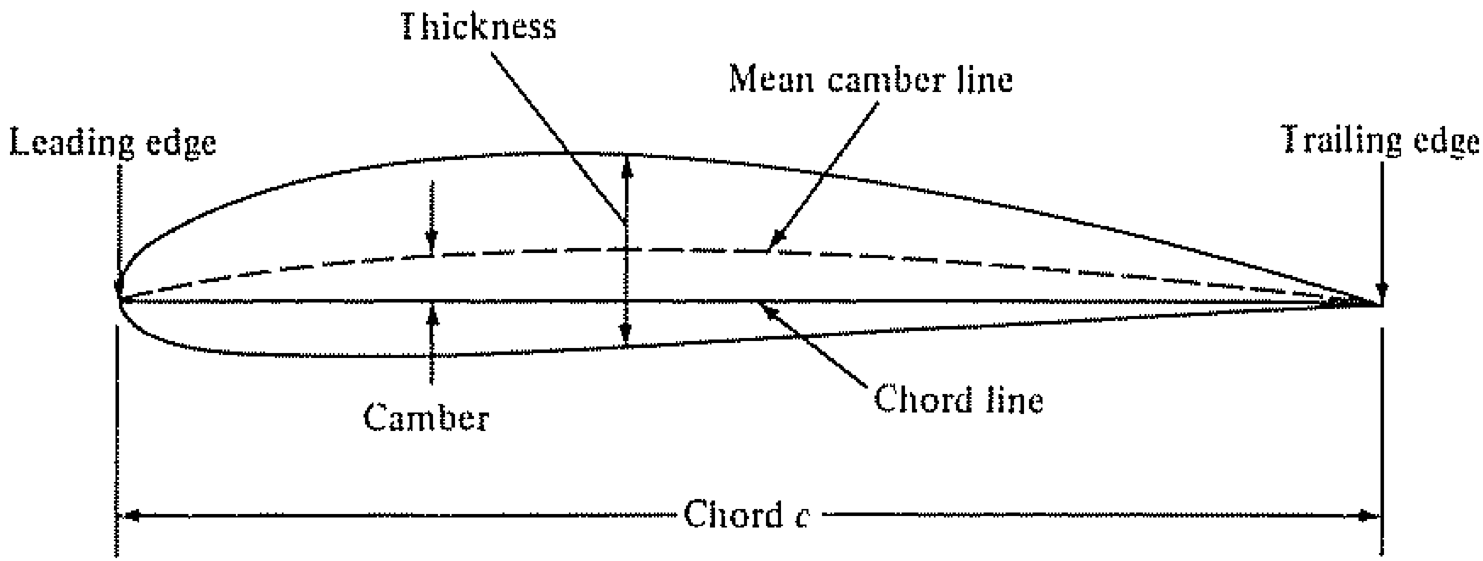
$\Gamma \neq 0$

- An ***airfoil*** is any section of the wing of airplane cut by a plane normal to y -axis





AIRFOIL NOMENCLATURE



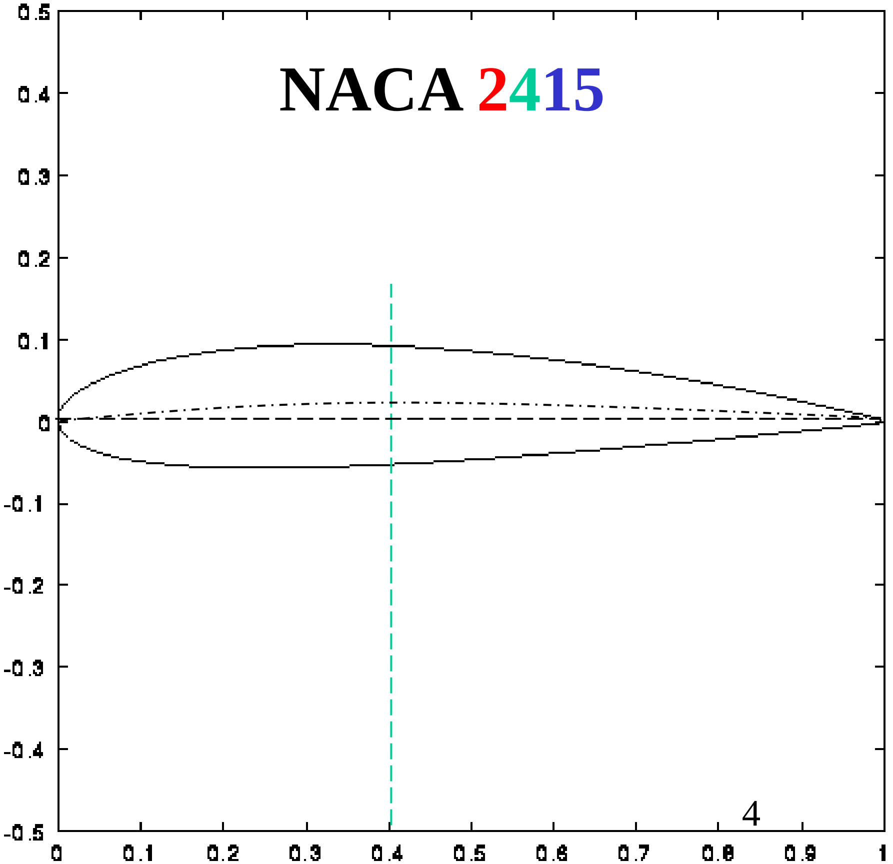
- **Mean Chamber Line:** Set of points halfway between upper and lower surfaces
  - Measured perpendicular to mean chamber line itself
- **Leading Edge:** Most forward point of mean chamber line
- **Trailing Edge:** Most reward point of mean chamber line
- **Chord Line:** Straight line connecting the leading and trailing edges
- **Chord, c:** Distance along the chord line from leading to trailing edge
- **Camber:** Maximum distance between mean chamber line and chord line
  - Measured perpendicular to chord line

NACA FOUR-DIGIT SERIES

- **First digit** specifies maximum camber in percentage of chord
- **Second digit** indicates position of maximum camber in tenths of chord
- **Last two digits** provide maximum thickness of airfoil in percentage of chord

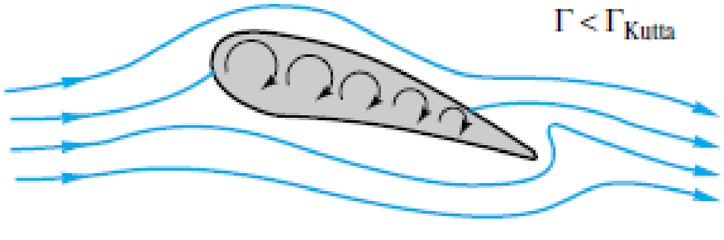
Example: **NACA 2415**

- Airfoil has maximum thickness of **15%** of chord (0.15c)
- Camber of **2%** (0.02c) located **40%** back from airfoil leading edge (0.4c)



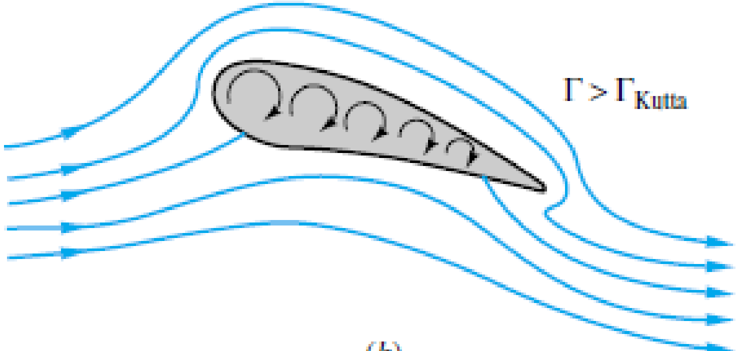
Statement of Kutta conditions:

a) If the value of Γ, too little circulation, stagnation point on rear upper surface;



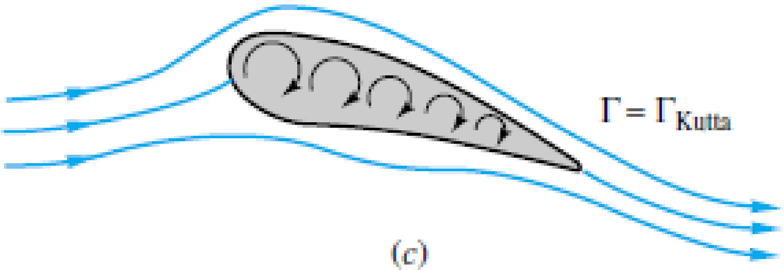
(a)

b) If the value of Γ, is too much, stagnation point at rear lower surface;



(b)

c) ***Just the right Kutta condition, smooth flow at TE.***



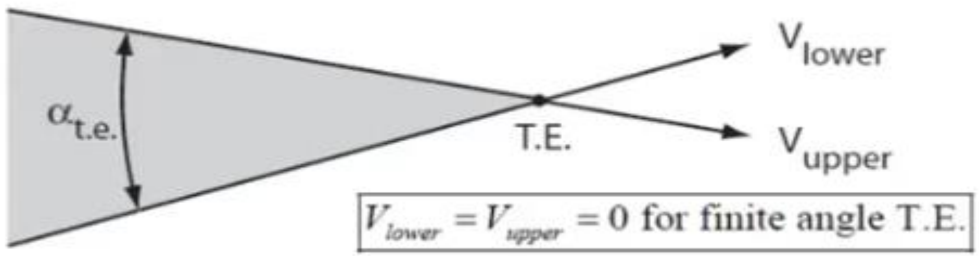
(c)

- Different possible shape of the trailing edge and their relation to the Kutta condition.

$$p_{t.e.} = p_\infty + \frac{1}{2}\rho V_\infty^2$$

TE is a stagnation point with $p_{t.e.} \equiv$ total pressure

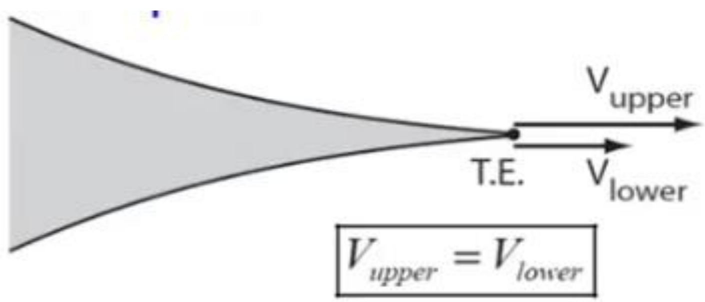
➢ If the TE angle is finite then the TE is stagnation point



Upper and lower surface velocities must still be tangent to their respective surfaces.

This implies 2 different velocities at TE.

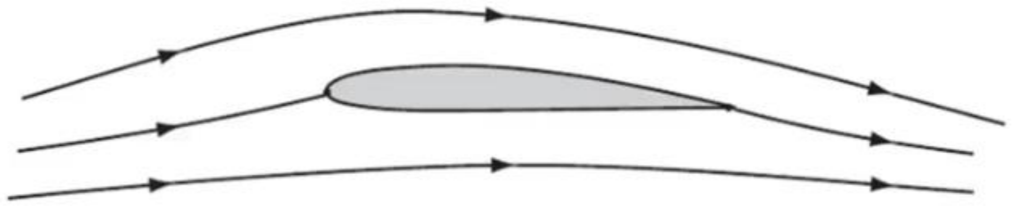
➢ If the trailing edge is cusped, the velocity leaving the top and bottom surface at the TE are finite and equal;



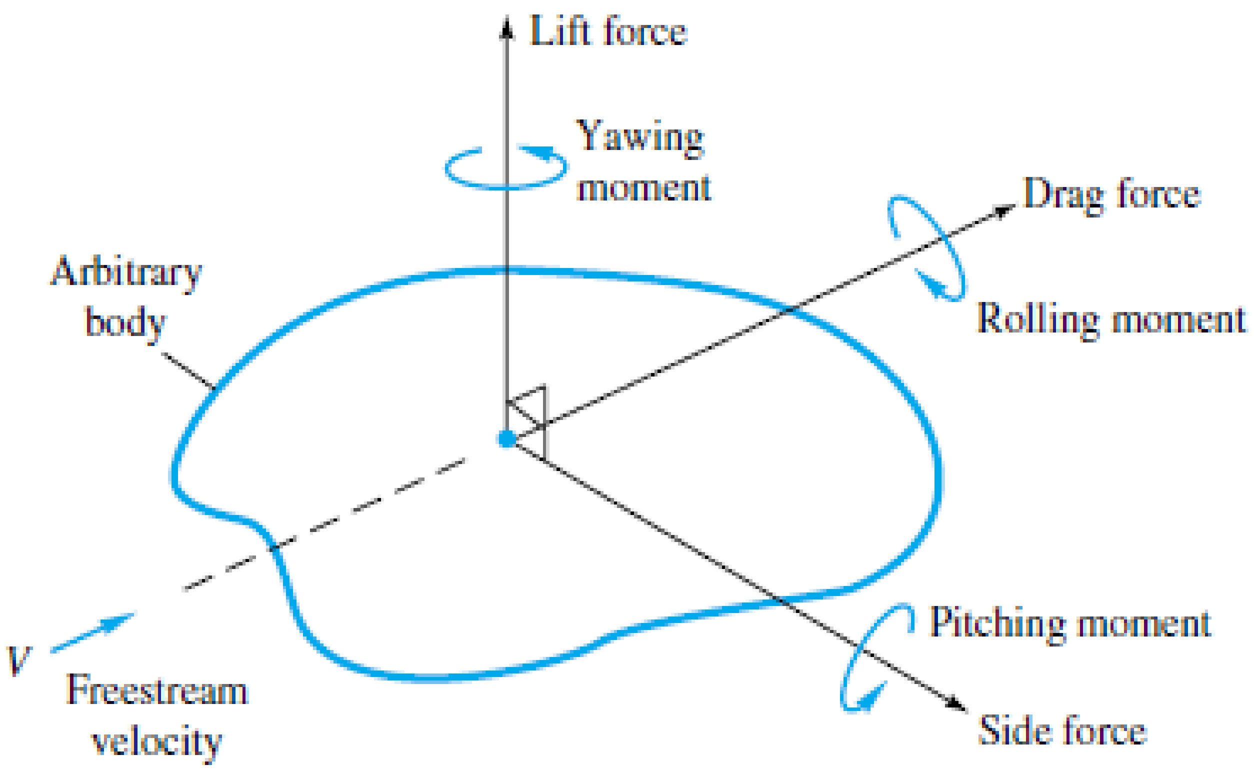
In this case, velocities from upper and lower surface are aligned.

Expression in terms of γ $\quad \gamma(TE) = 0$

- If the value of Γ, around the airfoil is such that the flow leaves the trailing edge smoothly.



- Any body of any shape when immersed in a fluid stream (uniform flow) will experience forces and moments from the flow.
- If the body has arbitrary shape and orientation, the flow will exert forces and moments about all three coordinate axes, as shown in Fig.

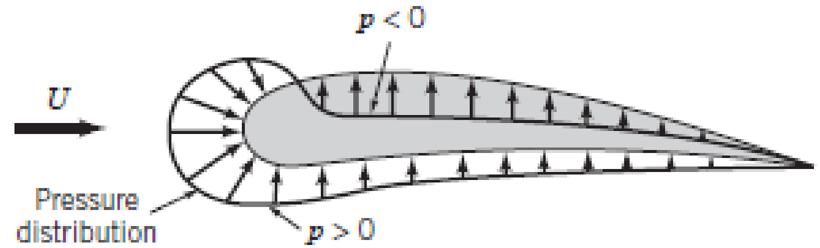


- The ***force on the body along this axis*** is called ***drag***, and the moment about that axis the ***rolling moment***.
    - The drag is essentially a flow loss and must be overcome if the body is to move against the stream.
- A second and very important ***force is perpendicular to the drag*** and usually performs a useful job, such as bearing the weight of the body. It is called the ***lift.***
    - The moment about the lift axis is called ***yaw.***
- The third component, ***neither a loss nor a gain***, is the ***side force***, and about this axis is the ***pitching moment***.

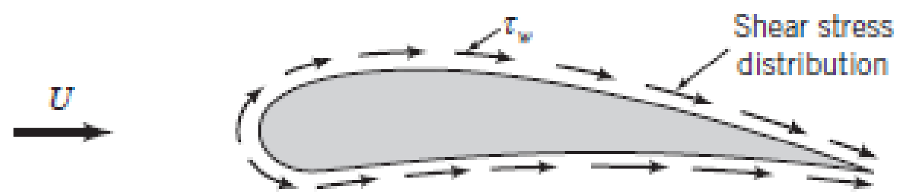
Lift and Drag Concept

- When any body moves through a fluid, an interaction between the body and the fluid occurs; this effect can be given in terms of the forces at the fluid–body interface.
- These forces Aerodynamic forces exerted by airflow comes from only two sources:

1. **Pressure, p**, distribution on surface
    - Acts **normal** to surface

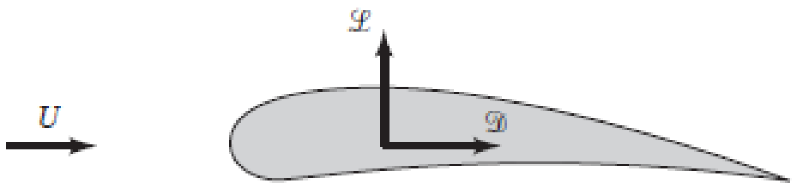


2. **Shear stress, τ_w**, (friction) on surface
    - Acts **tangentially** to surface



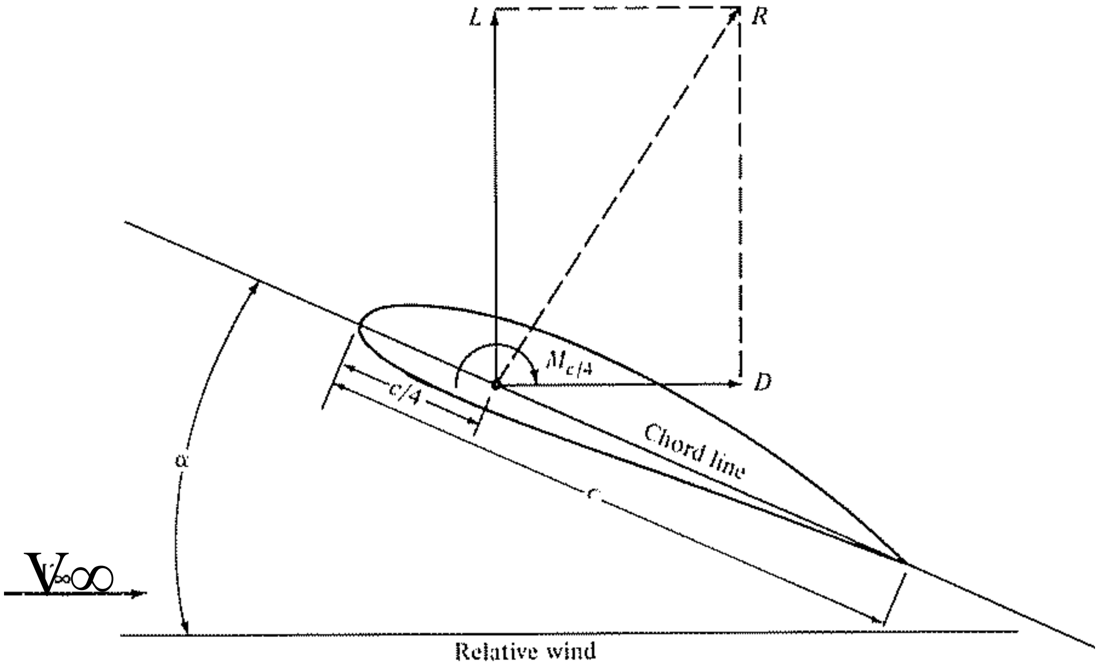
- Pressure and shear are in units of force per unit area (N/m^2)
- Net unbalance creates an aerodynamic force

- *"No matter how complex the flow field, and no matter how complex the shape of the body, the only way nature has of communicating an aerodynamic force to a solid object or surface is through the pressure and shear stress distributions that exist on the surface."*
- *"But, generally in the case of blunt bodies, the force will neither be parallel nor perpendicular to the surface. The force can be resolved into two components one parallel to the flow and the other perpendicular to the flow. The former may be called shear force and the other, the pressure force.*

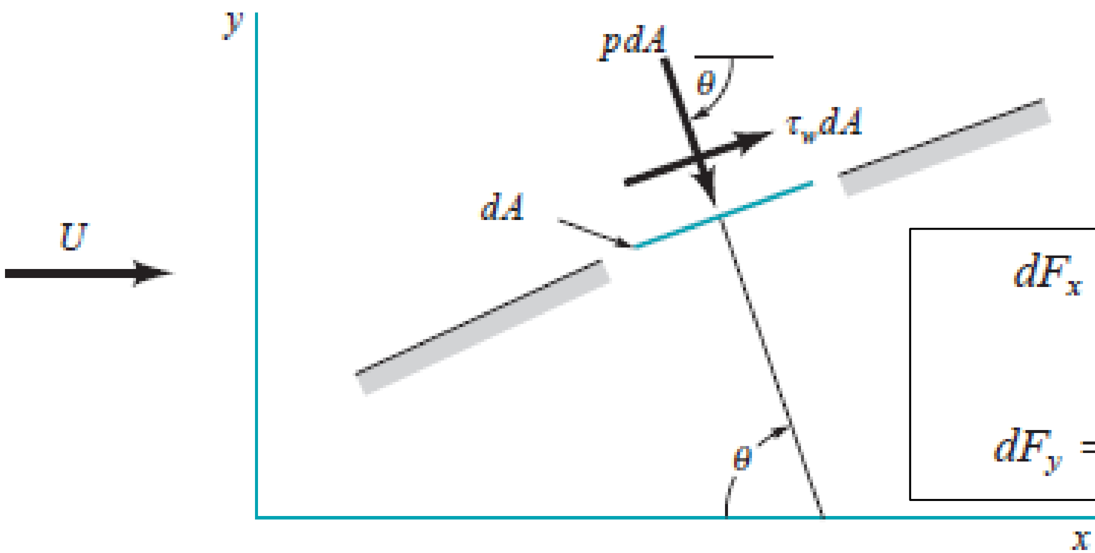


Resolving the Aerodynamic Force

- **Relative Wind:** Direction of V_∞
  - We use subscript ∞ to indicate far upstream conditions
- **Angle of Attack, α:** Angle between relative wind (V_∞) and chord line
- **Center of pressure**: The point on the body about which the aerodynamic moment is zero.
- Total aerodynamic force, **R**, can be resolved into two force components
  – **Lift, L:** Component of aerodynamic force **perpendicular** to relative wind
  – **Drag, D:** Component of aerodynamic force **parallel** to relative wind



- In Fig. **below,** let the pressure of fluid acting on a given small area dA on the body surface be p, and the friction force per unit area be τ_ω.
- The force pdA due to the pressure p acts normal to dA, while the force due to the friction stress, τ_ω acts tangentially.
- The *x* and *y* components of the fluid force on the small area element *dA* are:



$$dF_x = (p\,dA)\cos\theta + (\tau_w\,dA)\sin\theta$$

$$dF_y = -(p\,dA)\sin\theta + (\tau_w\,dA)\cos\theta$$

- Thus, the net x and y components of the force on the object are the sum of friction drag, dF_x and pressure drag, dF_y .

$$\mathscr{D} = \int dF_x = \int p \cos\theta \, dA + \int \tau_w \sin\theta \, dA$$

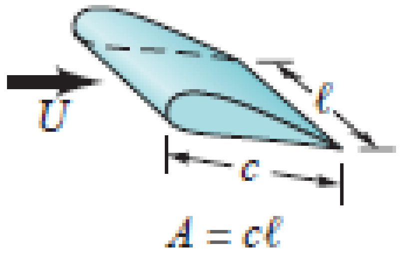
$$\mathscr{L} = \int dF_y = -\int p \sin\theta \, dA + \int \tau_w \cos\theta \, dA$$

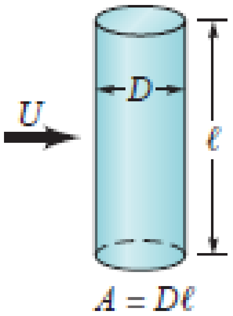
- **Drag Coefficient, C_D**
  - Drag is the component of force acting parallel to the direction of motion. Using the method of dimensional analysis the drag force can be related to flow Reynolds number by:

$$\frac{D}{\rho A V^2} = f(Re)$$

- Defining **coefficient of drag**, as the ratio of dynamic pressure it is seen that:

$$C_D = \frac{Drag}{1/2\rho A U^2}$$





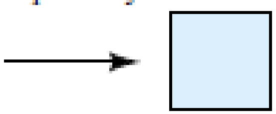
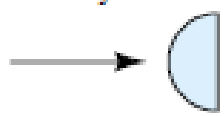
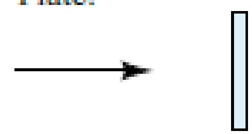
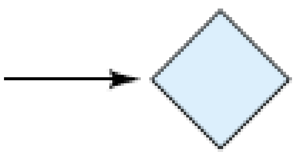
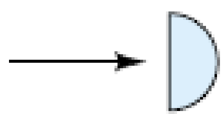
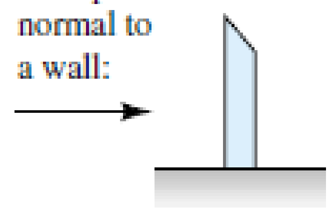
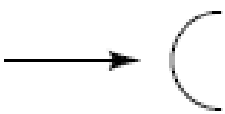
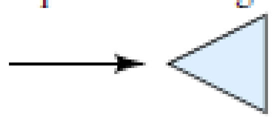
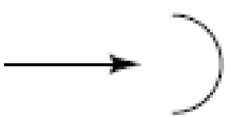
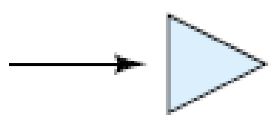
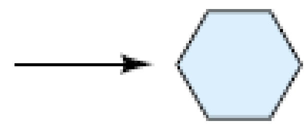
- **Lift coefficient,**

$$C_L = \frac{Lift}{1/2\rho A U^2}$$

Where. **A** is ***characteristics area*** which may differ depend up on the body shape.

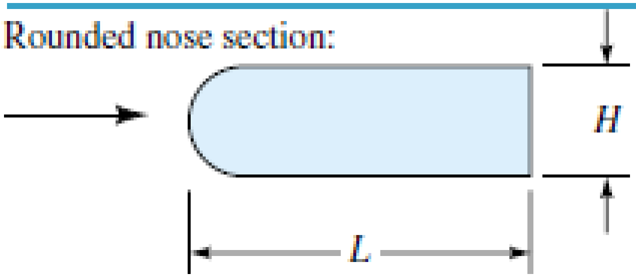
- ***Frontal area,*** the body as seen from the stream; suitable for thick, stubby bodies, such as spheres, cylinders, cars, missiles, projectiles, and torpedoes.
- ***Planform area***, the body area as seen from above; suitable for wide, flat bodies such as wings and hydrofoils.
- ***Wetted area,*** customary for surface ships and barges.

- **Table below,** gives a few data on drag, based on **frontal area**, of two dimensional bodies of various cross section, at Re ≥ 10^4.

Shape	C_D based on frontal area	Shape	C_D based on frontal area	Shape	C_D based on frontal area
Square cylinder: 	2.1	Half-cylinder: 	1.2	Plate: 	2.0
	1.6		1.7	Thin plate normal to a wall: 	1.4
Half tube: 	1.2	Equilateral triangle: 	1.6		
	2.3		2.0	Hexagon: 	1.0 0.7

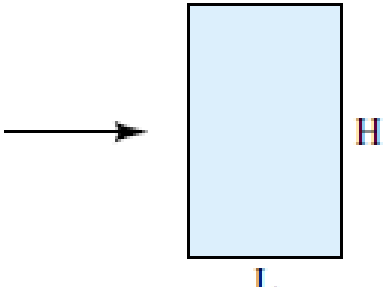
Shape	C_D based on frontal area

Rounded nose section:

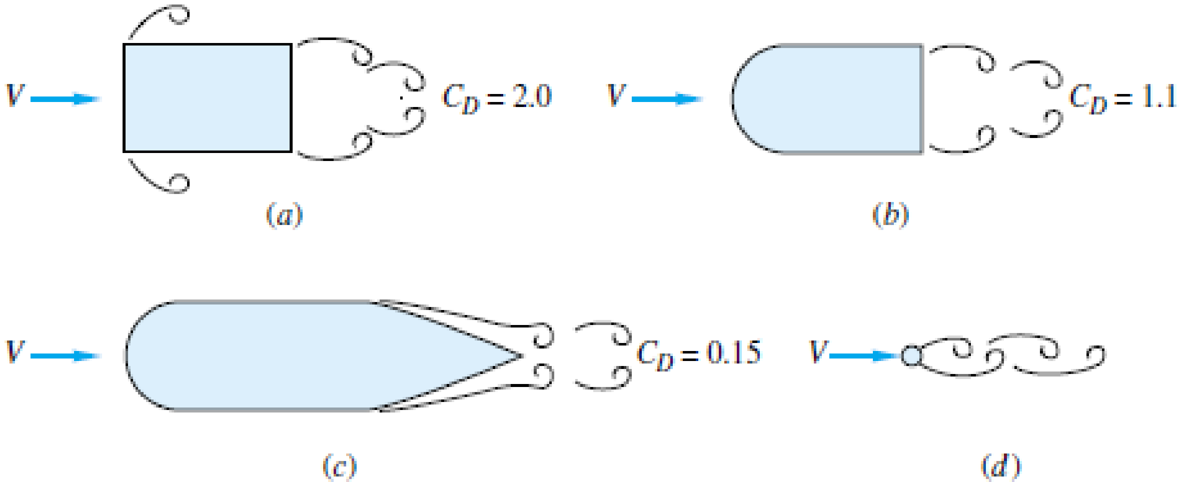


L/H:	0.5	1.0	2.0	4.0	6.0
C_D:	1.16	0.90	0.70	0.68	0.64

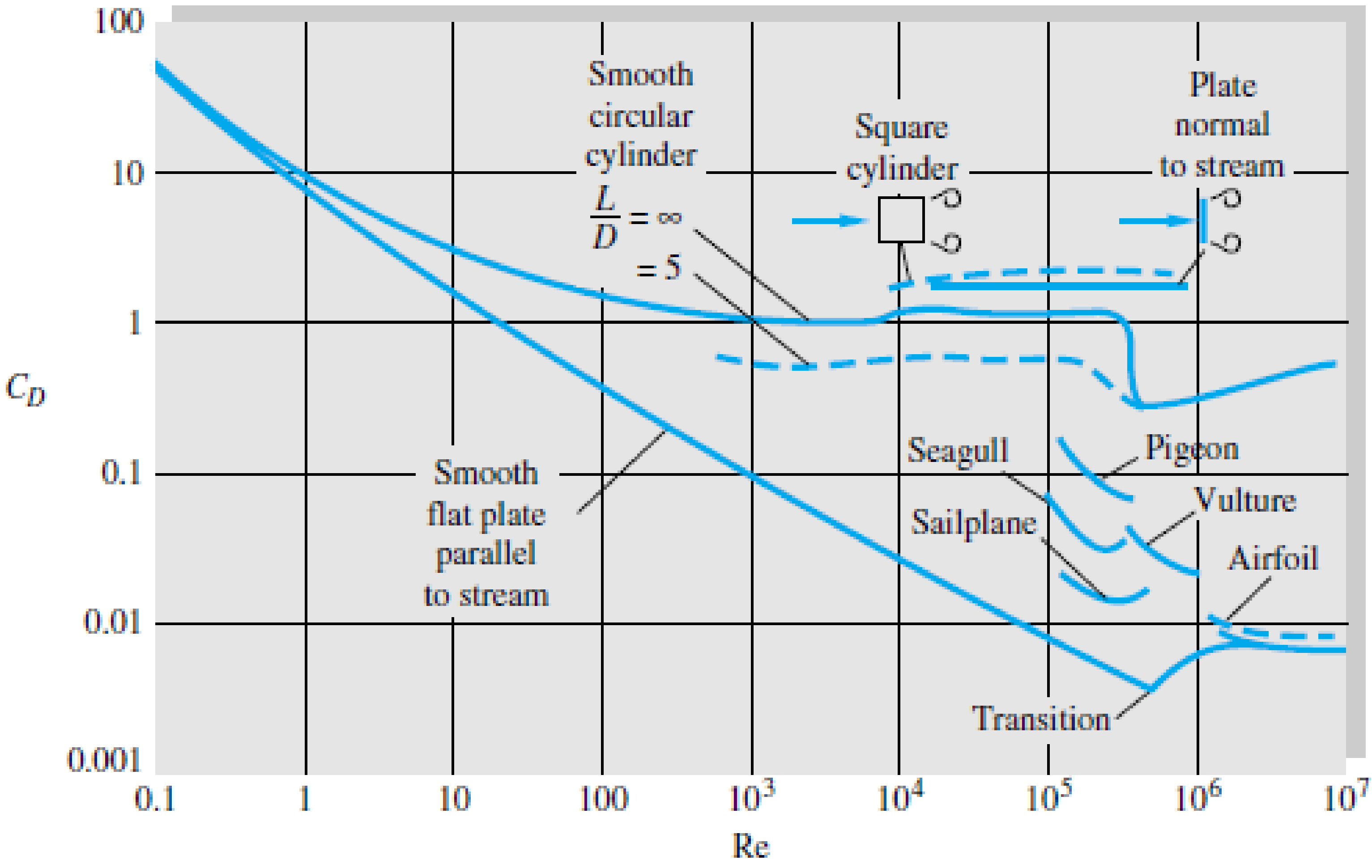
Flat nose section



L/H:	0.1	0.4	0.7	1.2	2.0	2.5	3.0	6.0
C_D:	1.9	2.3	2.7	2.1	1.8	1.4	1.3	0.9



- The drag of some representative **wide-span** (nearly two-dimensional) bodies is shown versus the Reynolds number in Fig.



- The historical trend for drag coefficients for automobiles and trucks.

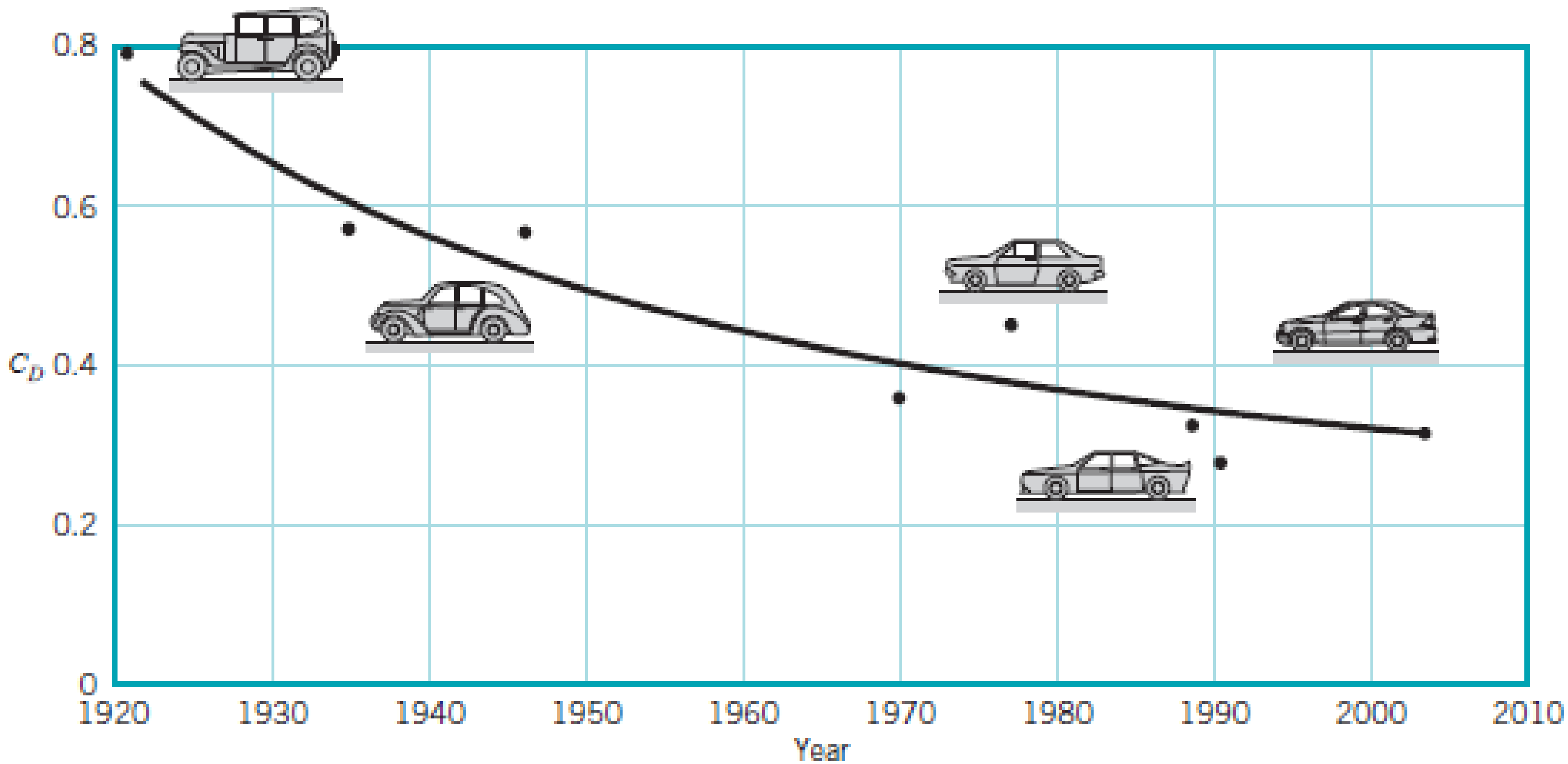


FIGURE 9.27 The historical trend of streamlining automobiles to reduce their aerodynamic drag and increase their miles per gallon (adapted from Ref. 5).

TRUCK SPOILER EXAMPLE

- Note ‘messy’ or turbulent flow pattern
- High drag
- Lower fuel efficiency

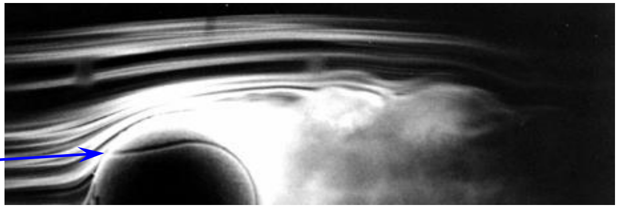
- Spoiler angle increased by + 5°
- Flow behavior more closely resembles a laminar flow
- Tremendous savings (< $10,000/yr)

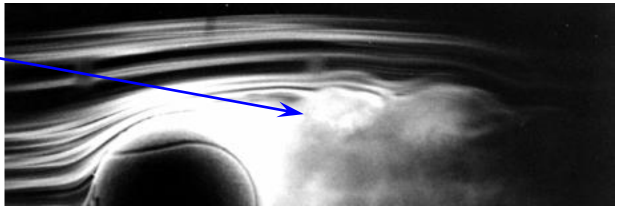
FLOW SEPARATION

- Key to understanding: Friction causes flow separation within boundary layer
- Separation then creates another form of drag called *pressure drag due to separation*

Friction (Viscous) Effects on Drag

- Friction has two effects:
  1. Skin friction due to shear stress at wall
  2. Pressure drag due to flow separation





$$D = D_{friction} + D_{pressure}$$

Total drag due to viscous effects = Drag due to skin friction + Drag due to separation

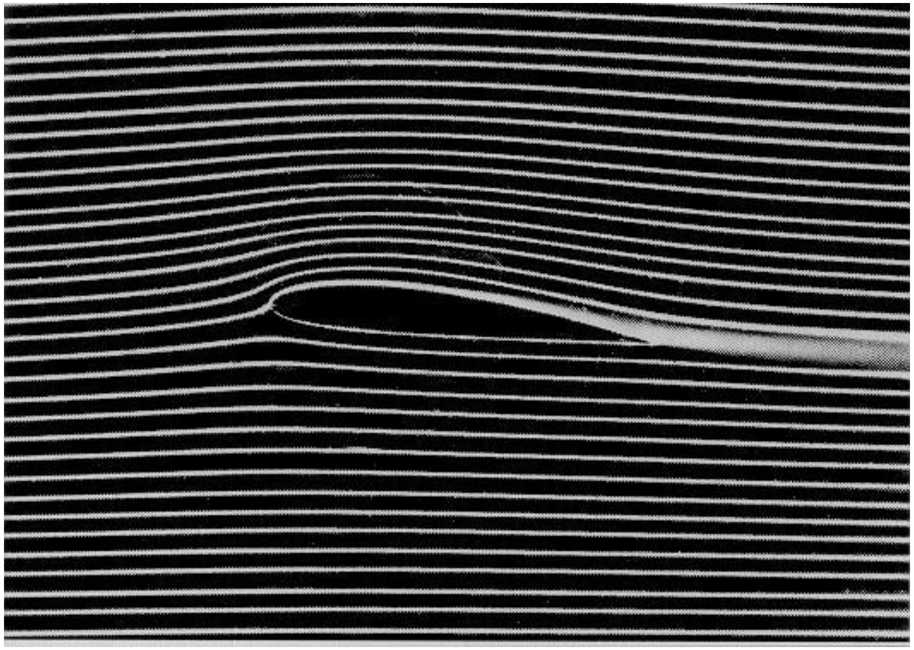
Called **Profile Drag**

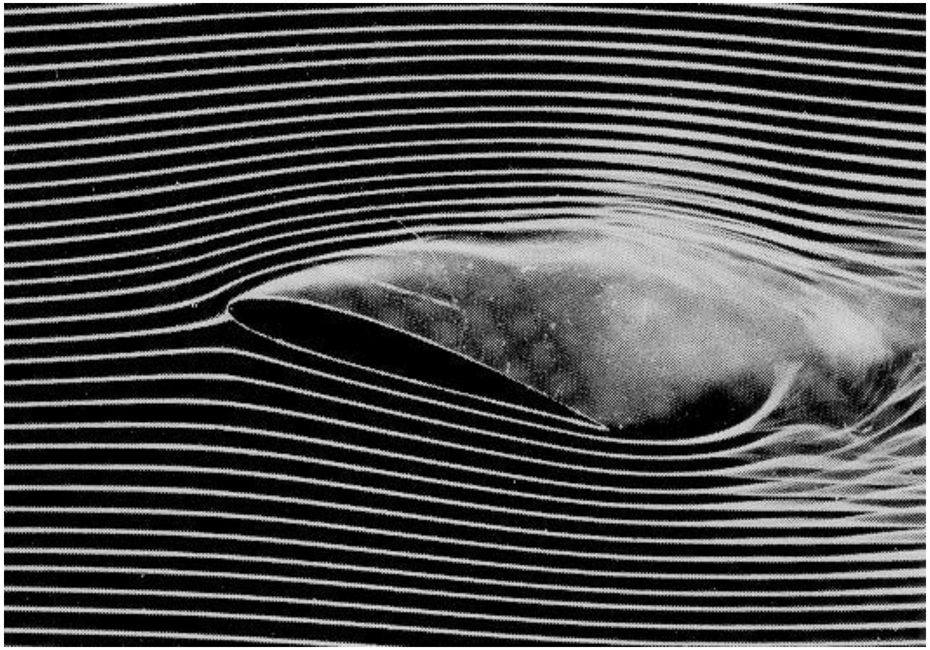
Drag due to skin friction ⇕ Less for laminar, More for turbulent

Drag due to separation ⇕ More for laminar, Less for turbulent

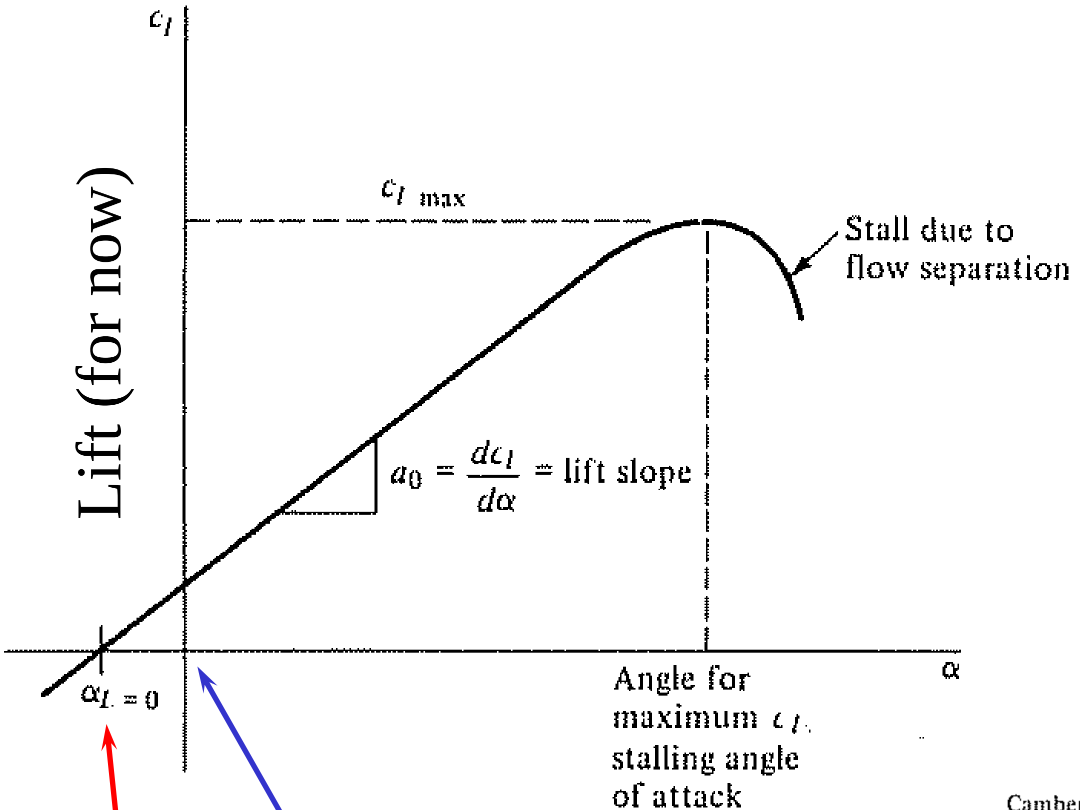
AIRFOIL STALL

- Key to understanding: Friction causes flow separation within boundary layer
  1. B.L. either laminar or turbulent
  2. All laminar B.L. → turbulent B.L.
  3. Turbulent B.L. 'fuller' than laminar B.L., more resistant to separation
- Separation creates another form of drag called pressure drag due to separation
  – Dramatic **loss of lift** and **increase in drag**





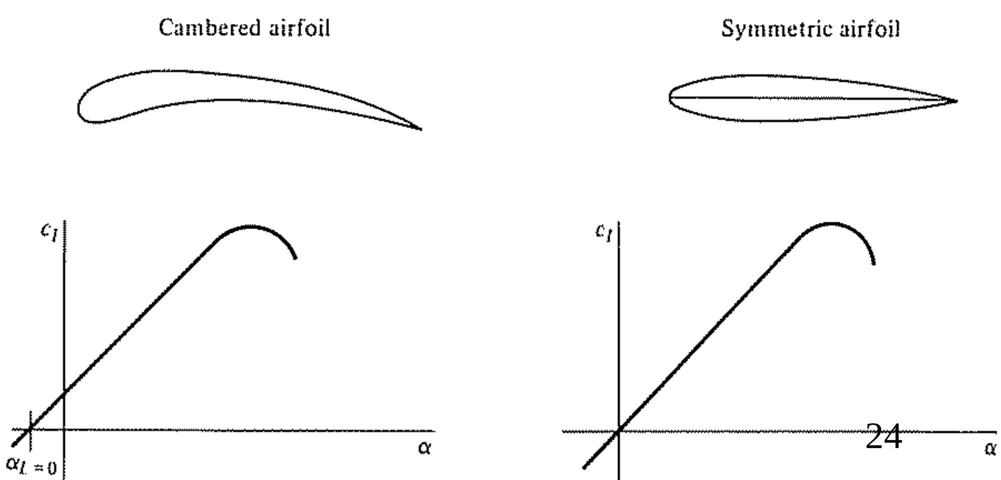
SAMPLE DATA



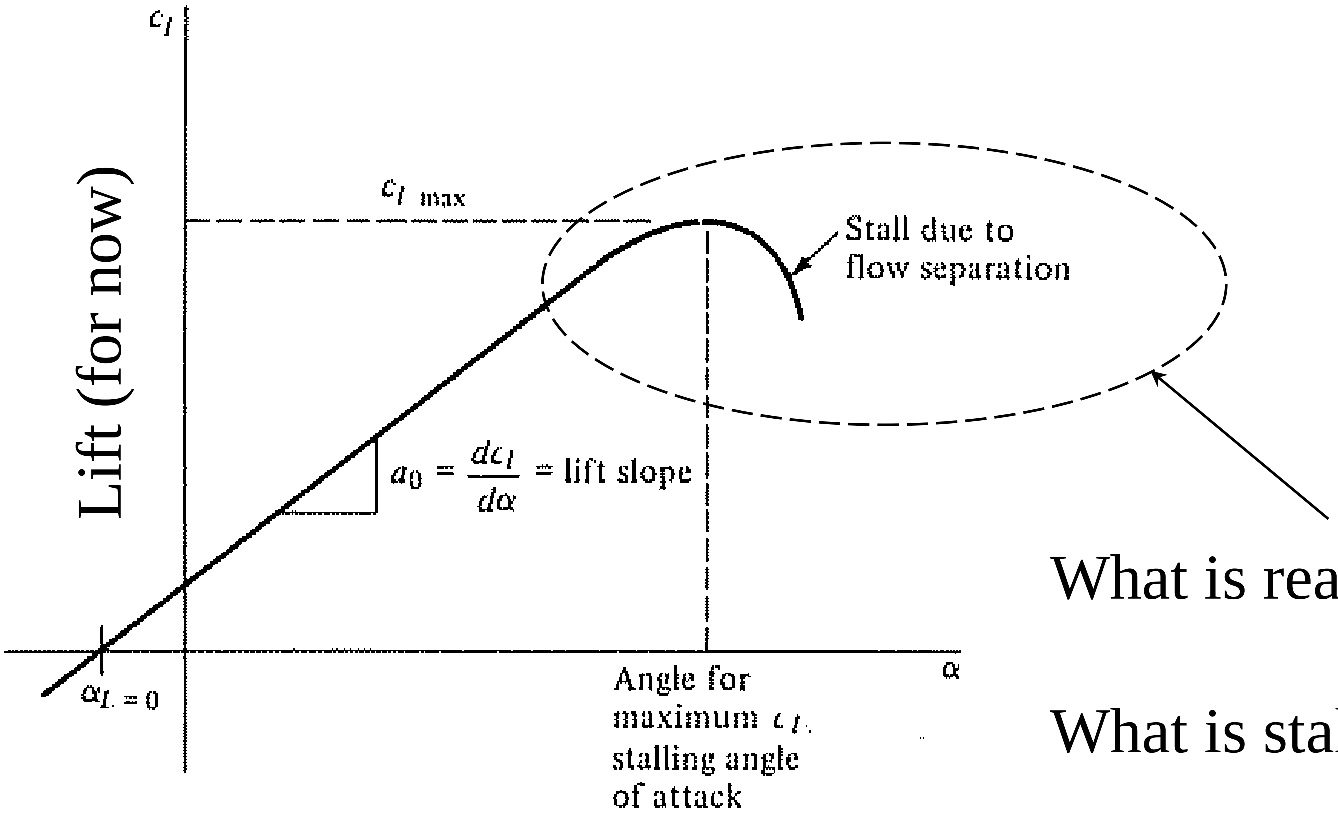
- Lift coefficient (or lift) linear variation with angle of attack, a
  - Cambered airfoils have positive lift when α = 0
  - Symmetric airfoils have zero lift when α = 0
- At high enough angle of attack, the performance of the airfoil rapidly degrades → **stall**

Cambered airfoil has lift at α=0

At negative α airfoil will have zero lift



SAMPLE DATA: STALL BEHAVIOR

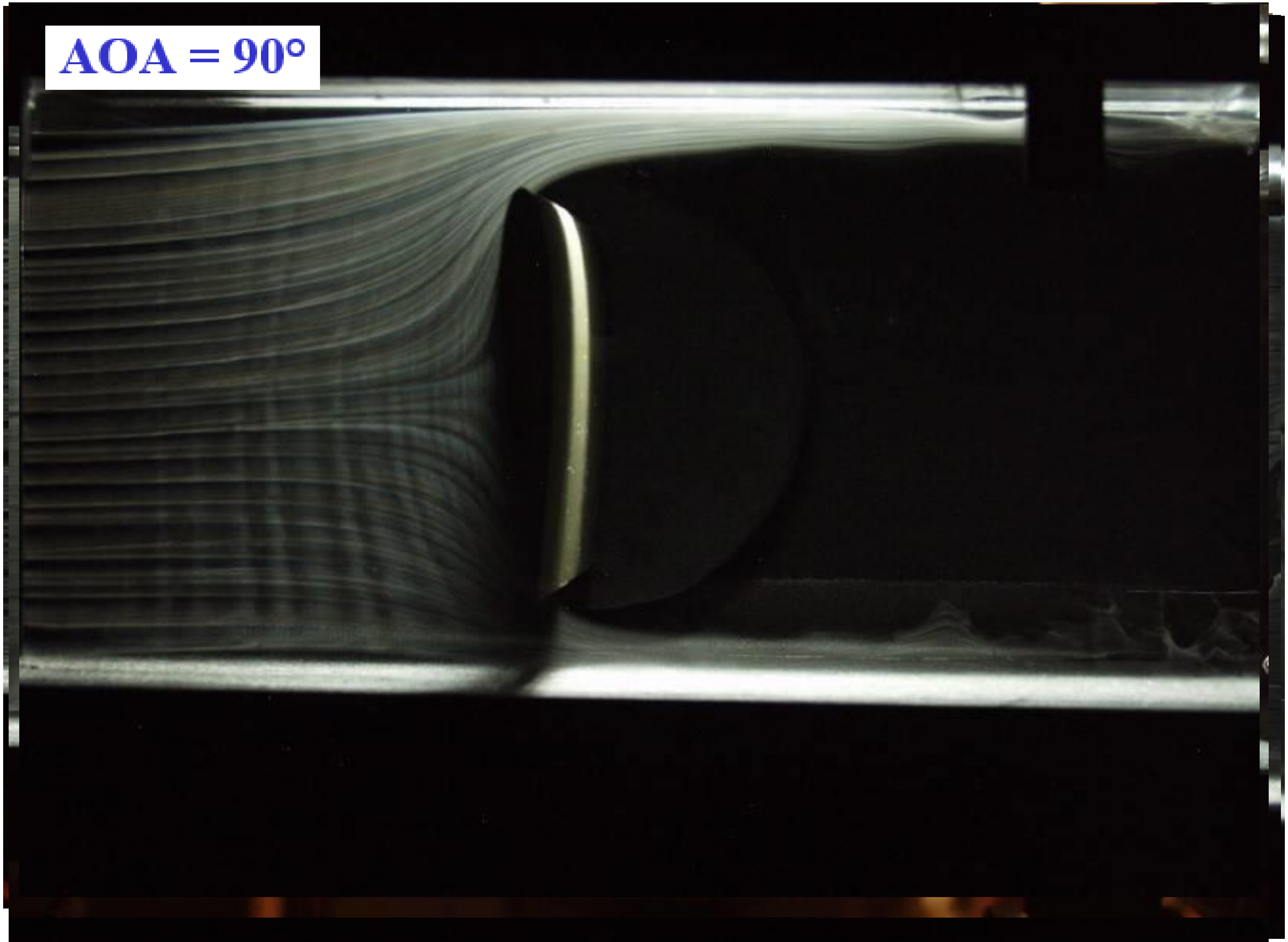


What is really going on here

What is stall?

Can we predict it?

Can we design for it?

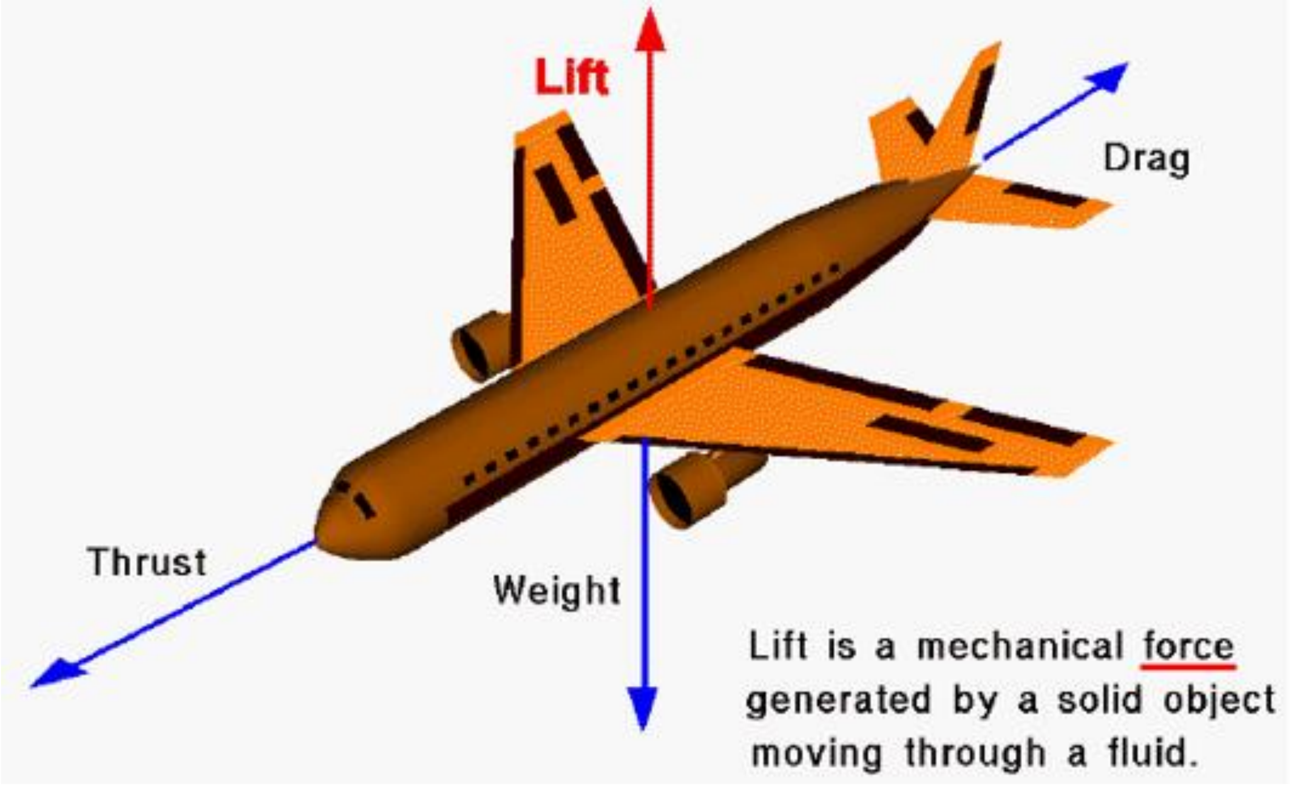

AOA = 90°

HOW DOES AN AIRFOIL GENERATE LIFT?

- Lift due to imbalance of pressure distribution over top and bottom surfaces of airfoil (or wing)
  - If pressure on top is lower than pressure on bottom surface, lift is generated
  - *Why* is pressure lower on top surface?

- We can understand answer from basic physics:
  - Continuity (Mass Conservation)
  - Newton's 2nd law (Euler or Bernoulli Equation)

HOW DOES AN AIRFOIL GENERATE LIFT?



Drag is the resistance an airplane experiences in moving forward through the air.

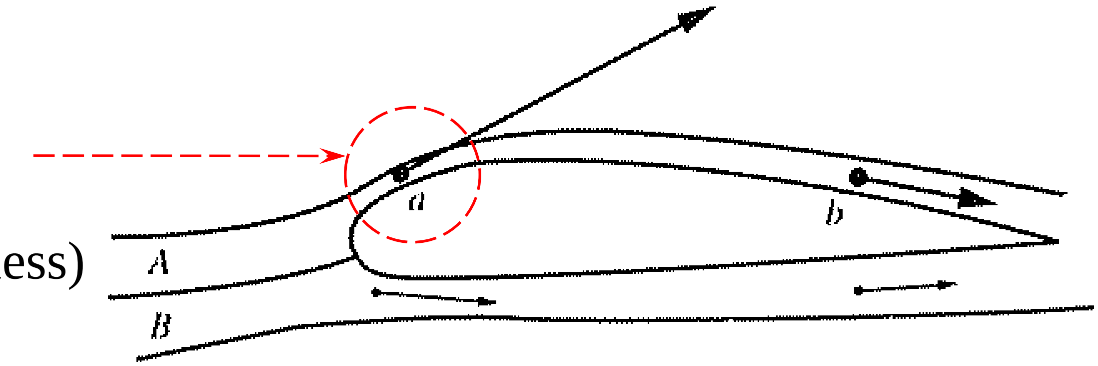
Or, it is the force a flowing fluid exerts on a body in *the flow direction*.

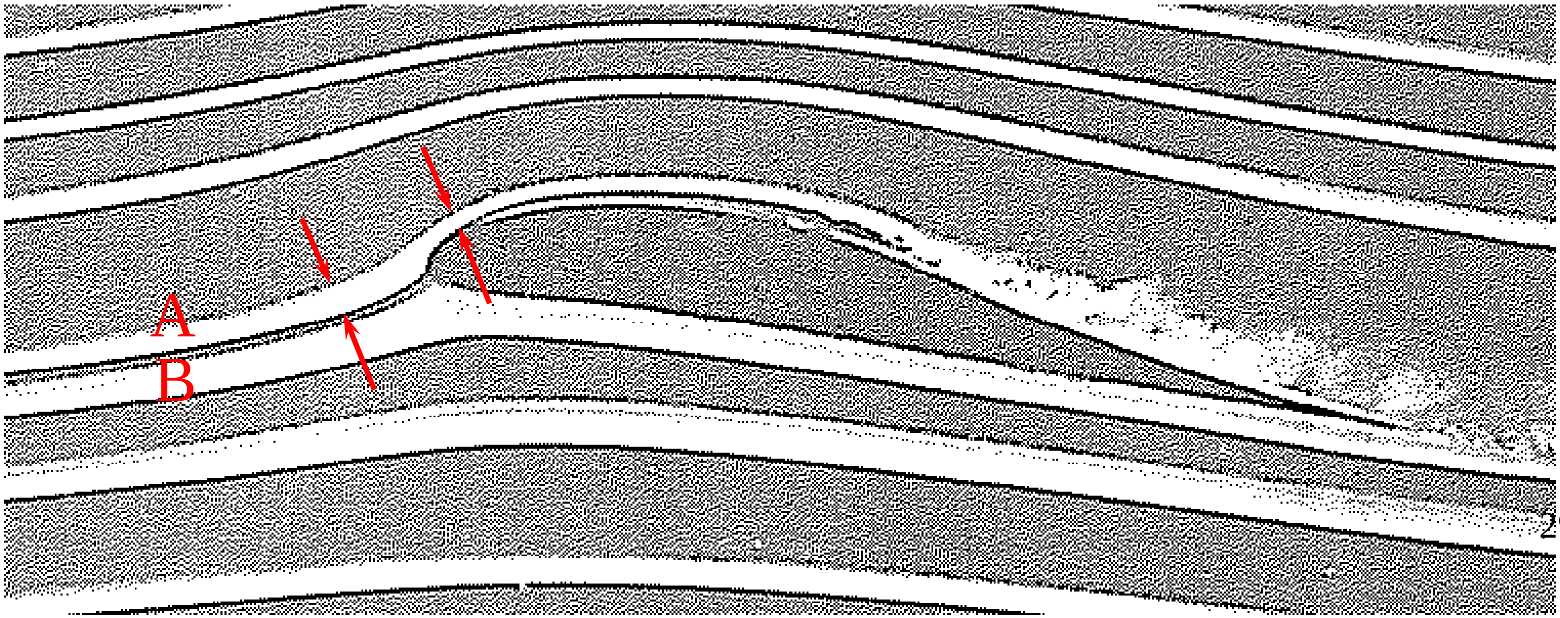
FIGURE 11–4
High winds knock down trees, power lines, and even people as a result of the drag force.

HOW DOES AN AIRFOIL GENERATE LIFT?

1. Flow velocity over top of airfoil is faster than over bottom surface
   - Stream tube A senses upper portion of airfoil as an obstruction
   - Stream tube A is squashed to smaller cross-sectional area
   - Mass continuity ρAV=constant: IF A↓ THEN V↑

Streamtube A is squashed most in nose region (ahead of maximum thickness)





HOW DOES AN AIRFOIL GENERATE LIFT?

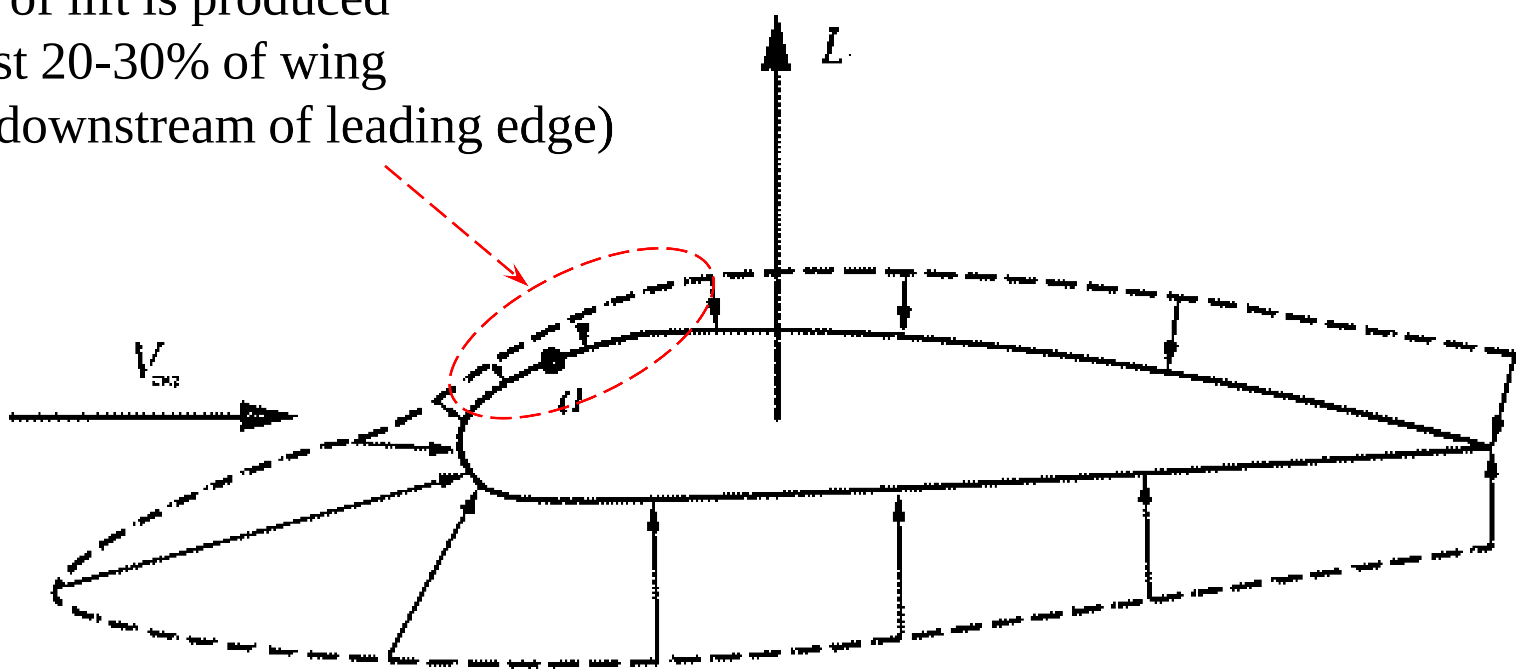
2. As V ↑ p↓
   – Incompressible: Bernoulli's Equation
   – Compressible: Euler's Equation
   – Called **Bernoulli Effect**

$$p + \frac{1}{2}\rho V^2 = \text{constant}$$

$$dp = -\rho V dV$$

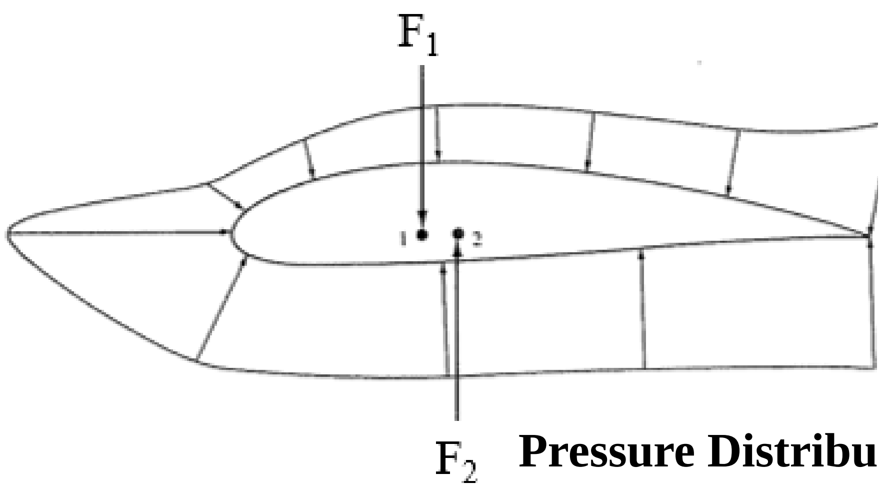
3. With lower pressure over upper surface and higher pressure over bottom surface, airfoil feels a net force in upward direction → Lift

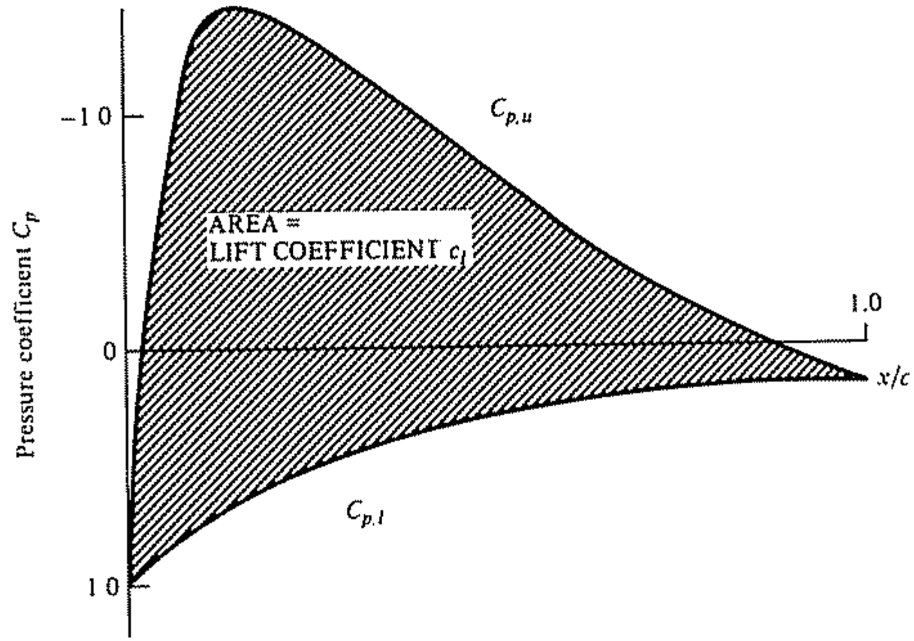
Most of lift is produced in first 20-30% of wing (just downstream of leading edge)

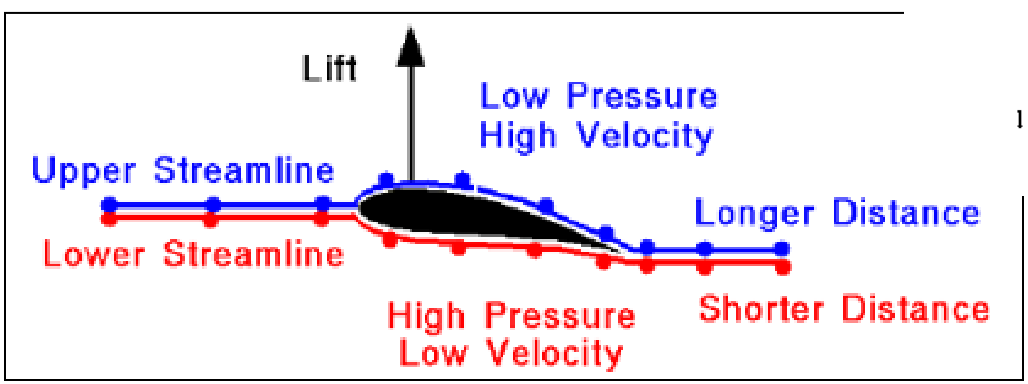


MORE DEFINITIONS

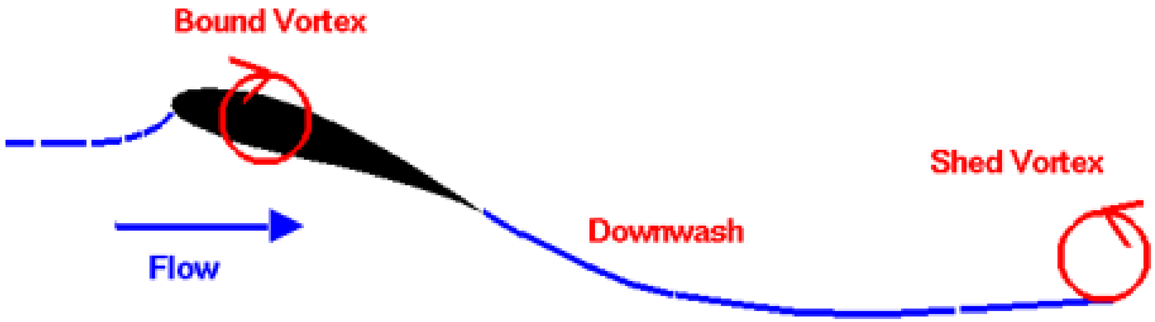
- Total aerodynamic force on airfoil is summation of F_1 and F_2
- Lift is obtained when $F_2 > F_1$
- Misalignment of F_1 and F_2 creates Moments, M, which tend to rotate airfoil/wing
  - A moment (torque) is a force times a distance
  - Pressure distribution in aerodynamic literature often given as C_p







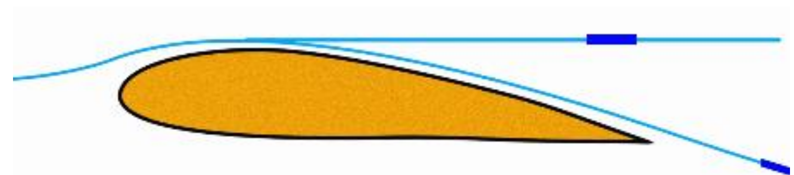
- An airfoil produce lift when moves in relative to air, that makes an airplane fly.
- **How does this lift produced?**
  - ✓ the airfoil produces a downwash as shown. This cause the pressure difference b/n the top and bottom of an airfoil and hence produce **lift.**



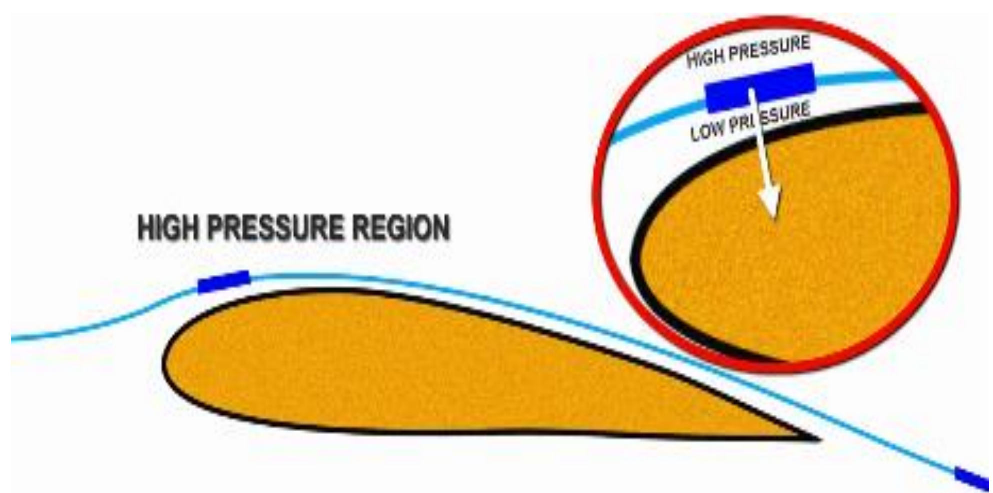
  - ✓ The higher the angle of attack the greater will be the down wash the airfoil lift force.
  - ✓ A greater air speed also increase the lift force significantly.

- **COANDA EFFECT:** the tendency of a fluid particles to get attached to a convex Surface or top airfoil section as shown.

- The particles approaches the airfoil and takes the curve as shown above. But, after the curve why doesn't move straight as shown.

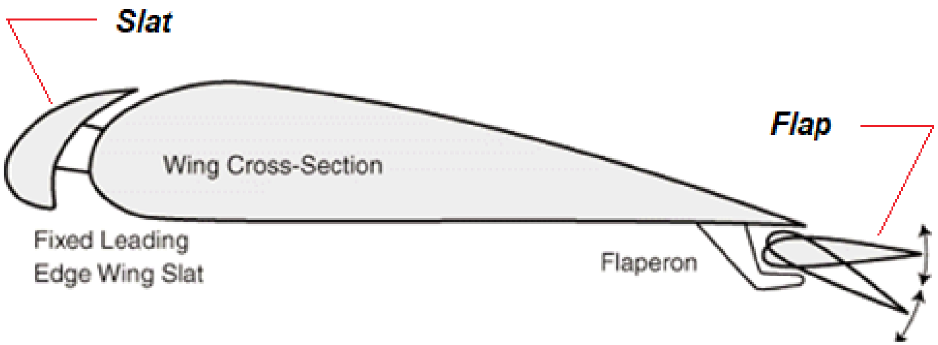


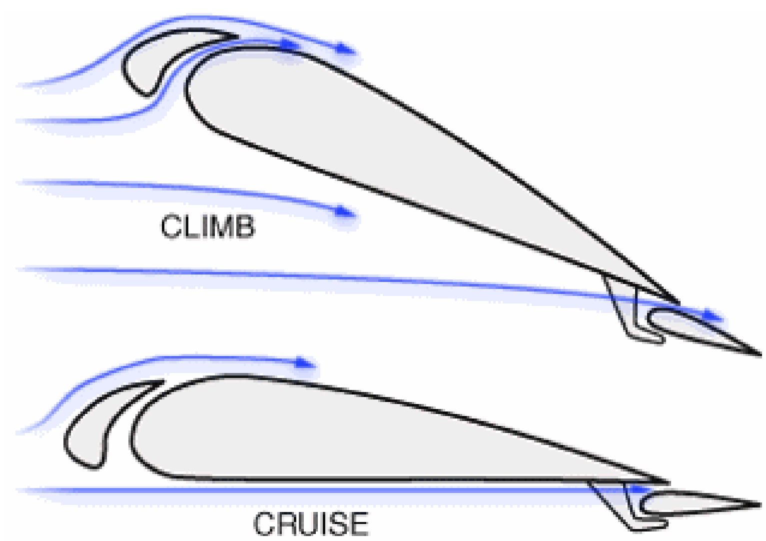
> - Examine this curve motion is more closely,
>   - ✓ In order to take the curve the particles have high pressure at the top and low pressure at bottom
>   - ✓ This supply the centrifugal force
>   - ✓ The higher the pressure push the particles downward – the effect is known as the Coanda effect.



Techniques to increase lift … cont'd

- High speed
- High angle of attack: to be the greater downwash
- Use (altering) of flap and slat – which increase downwash & wing area



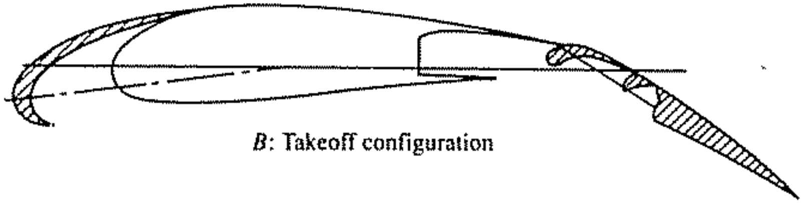


Some functions of airfoils:

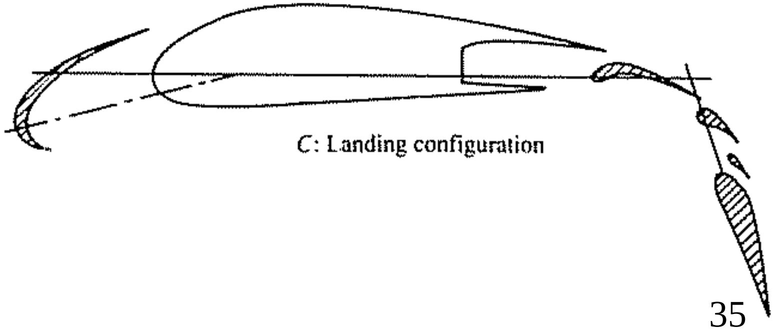
- Create lift
- Create thrust
- Attachment point for the control surfaces (including flaps, slots, slats, etc)
- House fuel tanks and instrument components
- Stability

EXAMPLE: BOEING 727 FLAPS/SLATS

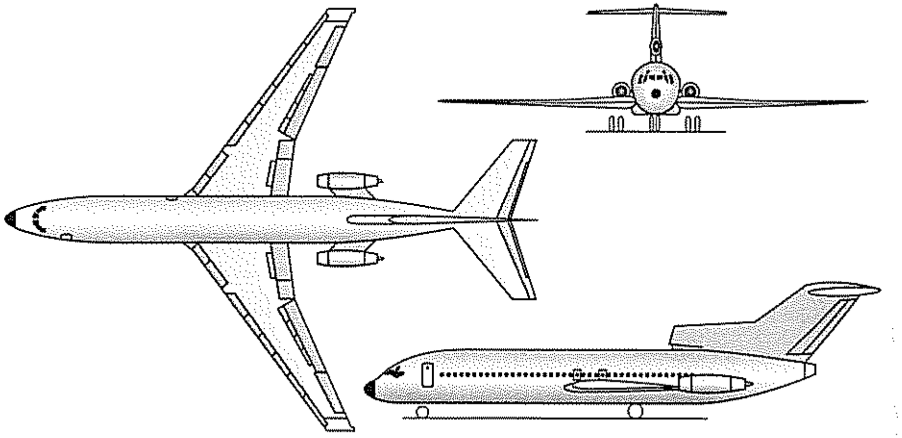
A: Cruise configuration



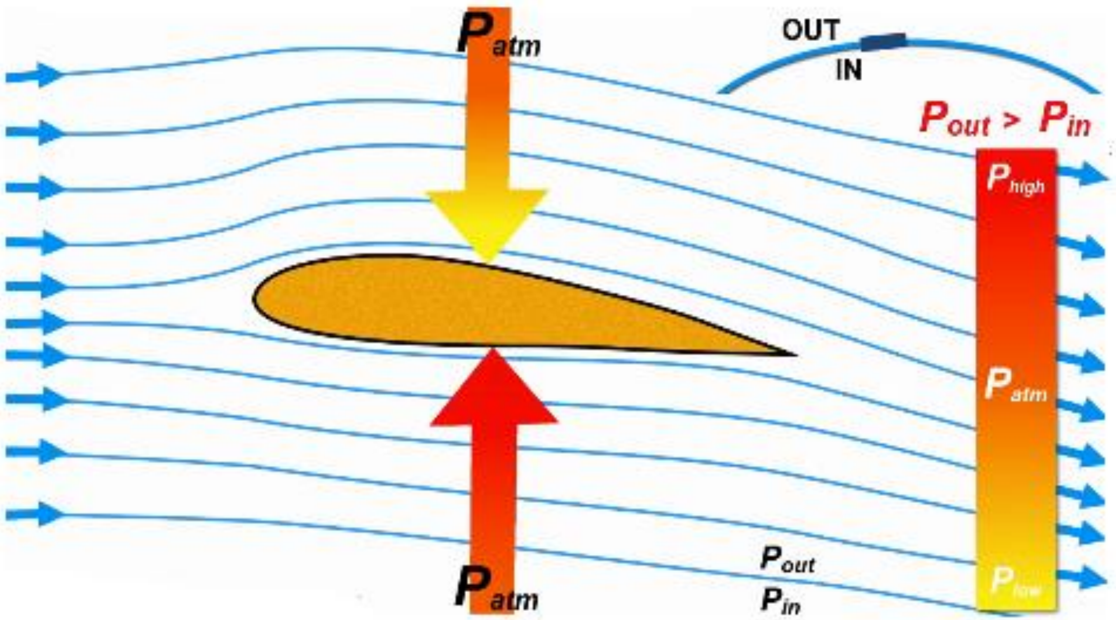
B: Takeoff configuration

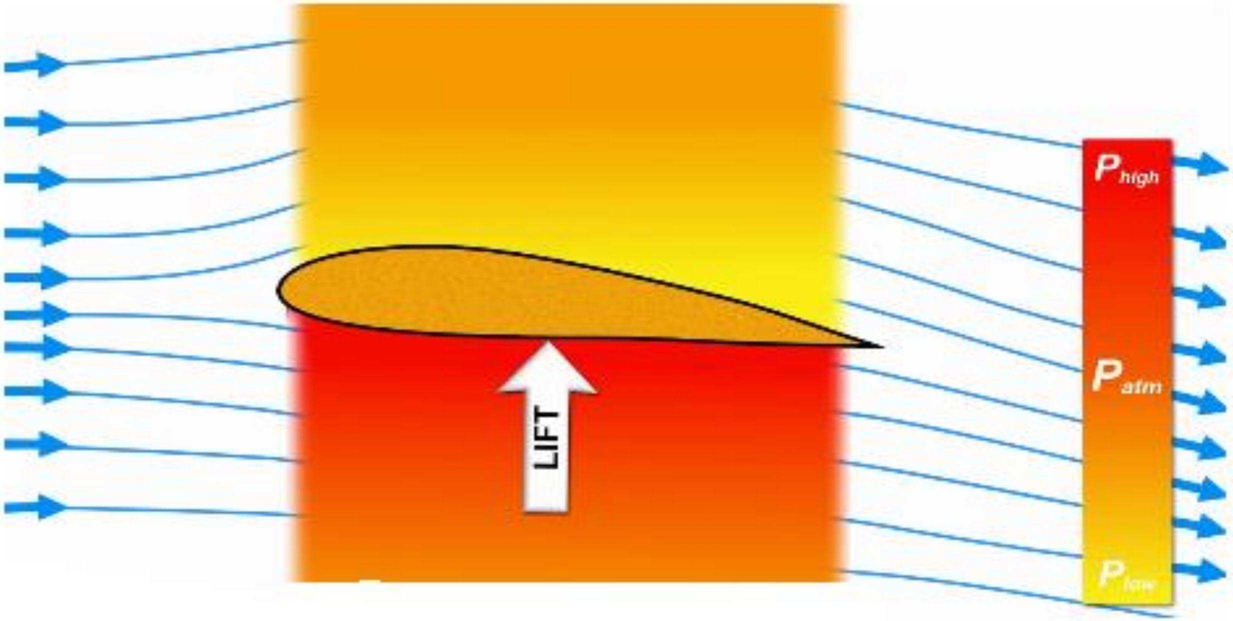


C: Landing configuration



Summary on lift

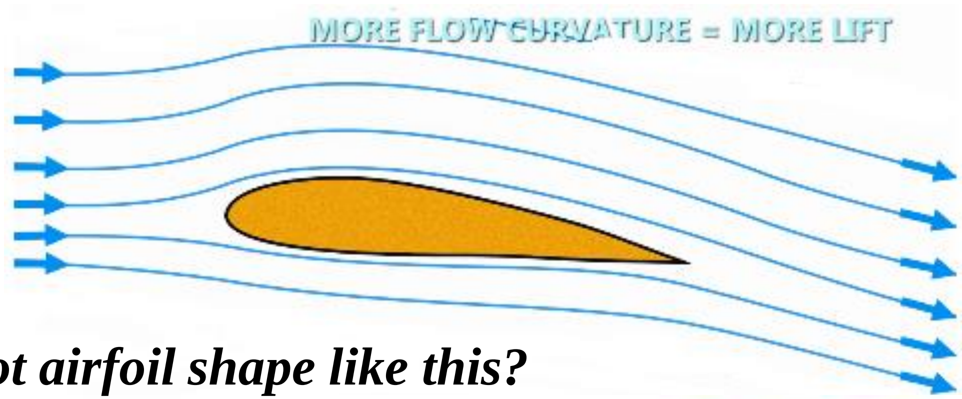




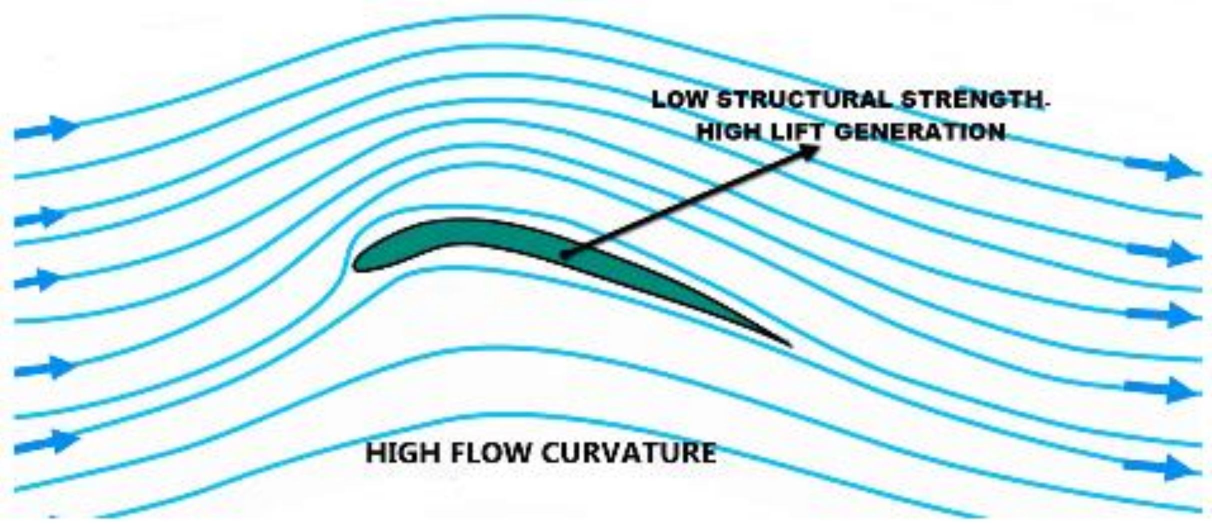
Summary on lift …..cont'd

- ❑ Basically the introduction of aerodynamics is about ***the flow curve.***
- ❑ ***This curvature generate pressure difference and the lift*** .

  - This means more curvature generate more lift.

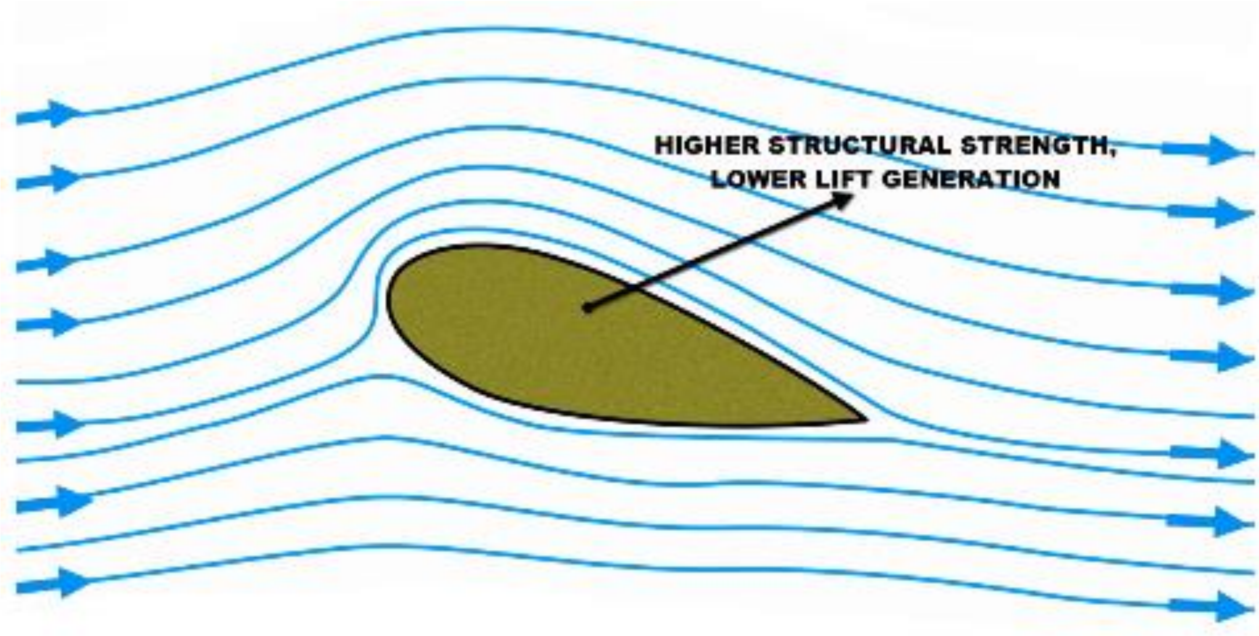


✓ ***But, why not airfoil shape like this?***



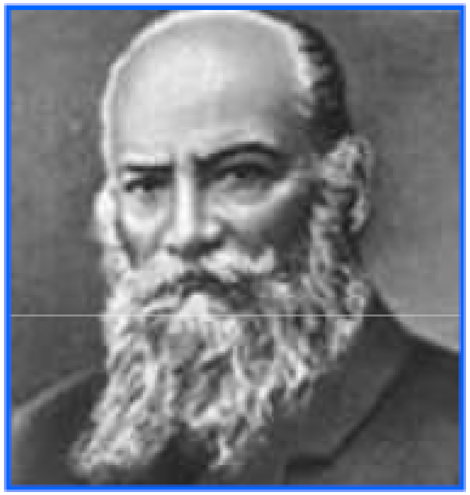
- The reason is to give structural support & space to accommodate the fuel takes

- ✓ ***What about this airfoil shaped?***

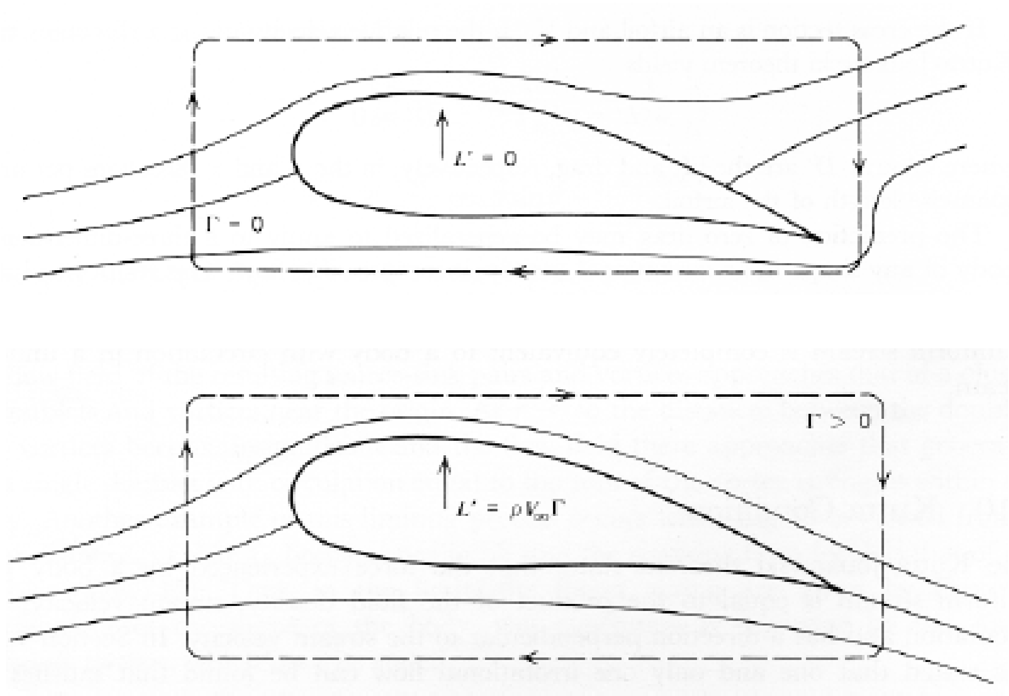


Finite Wing Theory

Kutta-Joukowsky Law,
Kutta-Joukowsky Condition



$$L^{'} = \rho V_{\infty} \Gamma$$

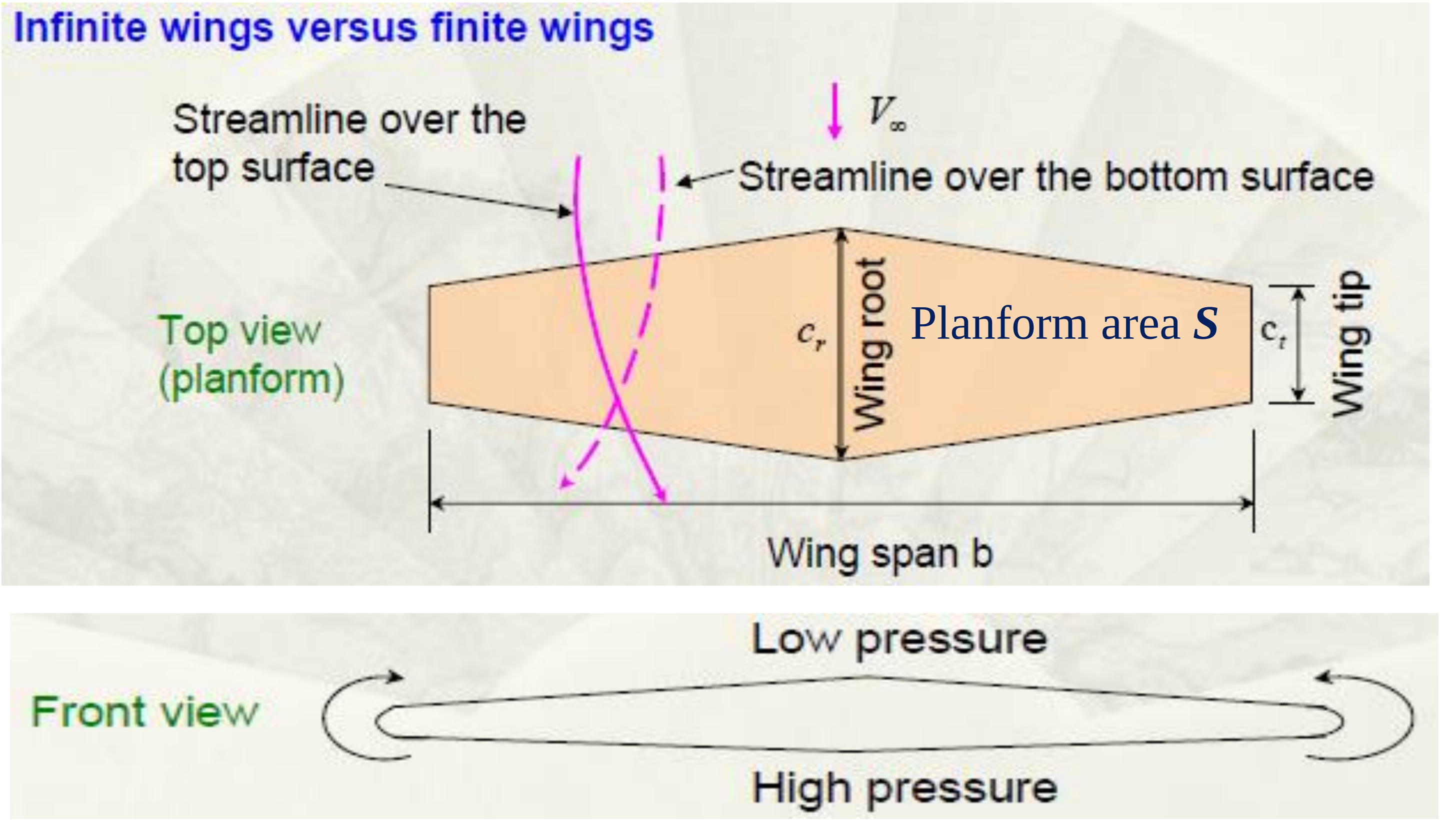


Finite Wing Theory

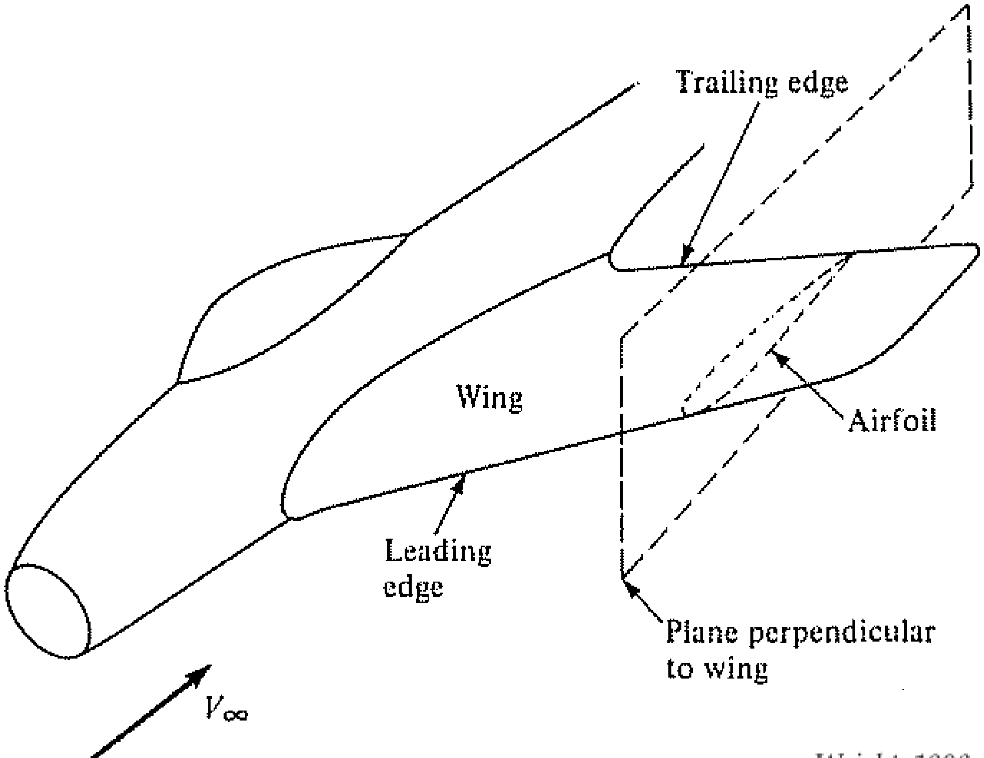
Lift for a Finite Wing

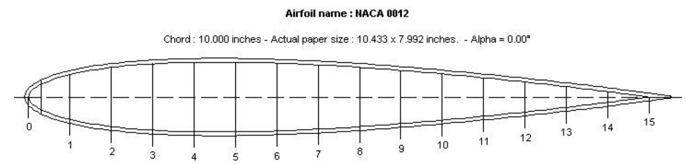
- All real wings are finite in span (airfoils are considered as infinite in the span)
- And, all real wings have ***tips*** and are therefore of finite span or finite aspect ratio AR, defined by:

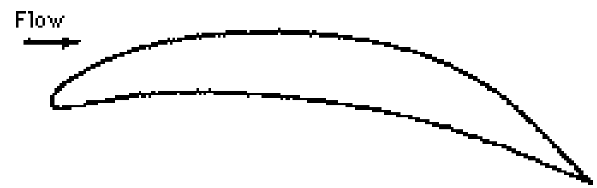
$$Aspect\ ratio\ AR = \frac{b^2}{S}\ \text{:Taper ratio} = \frac{C_t}{C_r}$$



AIRFOILS VERSUS WINGS



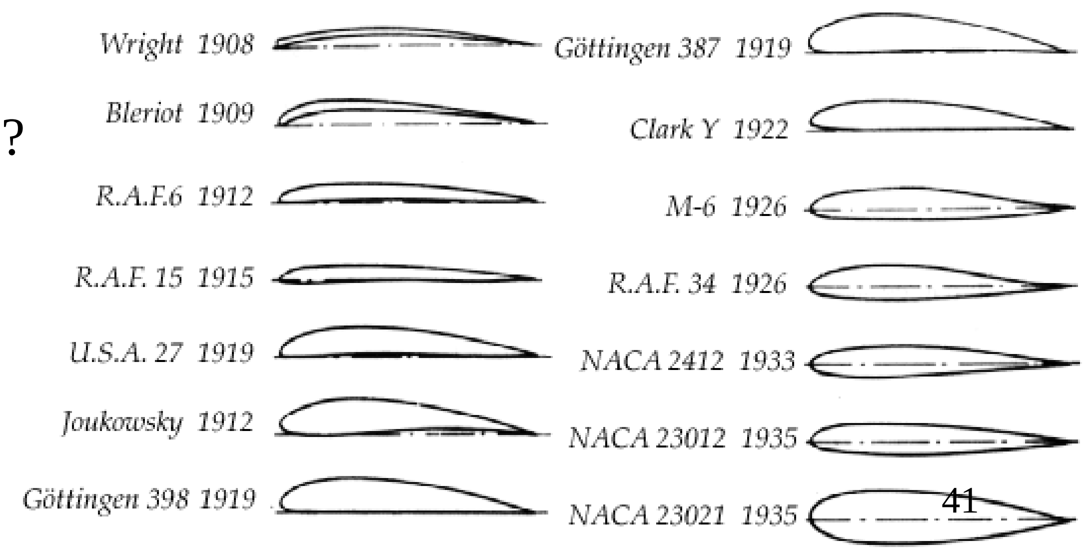




Why do airfoils have such a shape?

How are lift and drag produced?

NACA airfoil performance data



INFINITE VERSUS FINITE WINGS

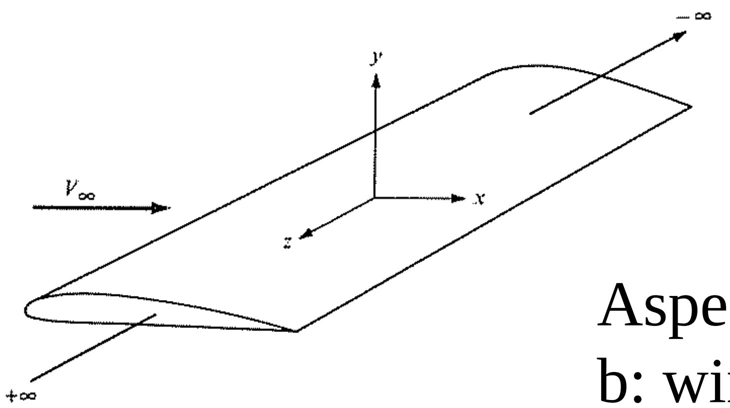
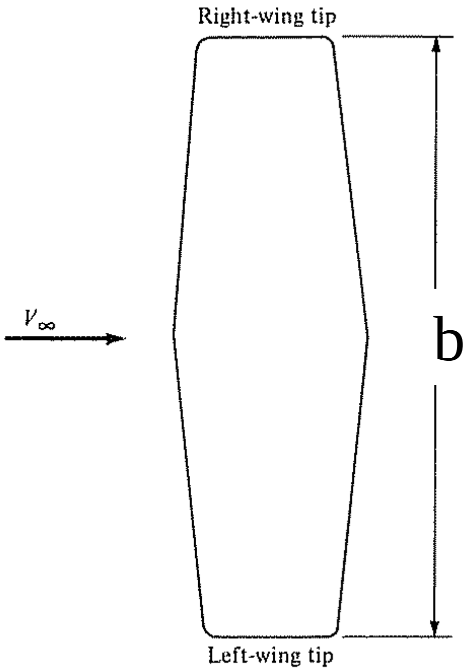


Figure 5.11 Infinite (two-dimensional) wing



Aspect Ratio
b: wingspan
S: wing area

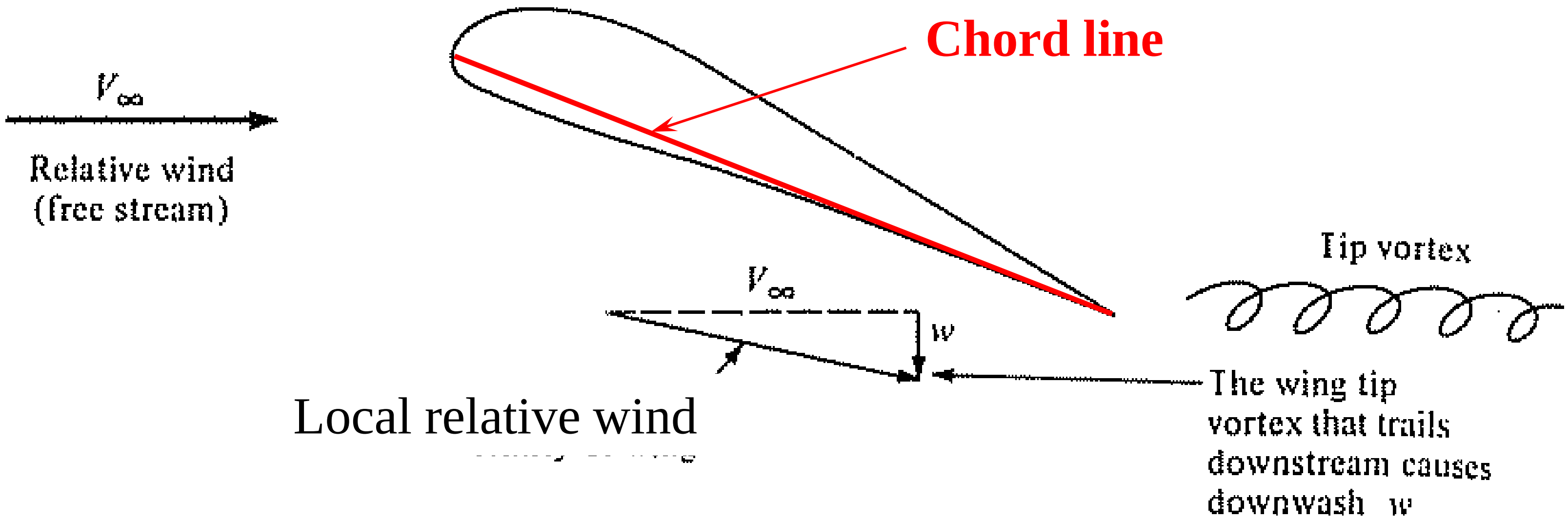
$$AR \equiv \frac{b^2}{S}$$

- Upper surface (upper side of wing): low pressure
- Lower surface (underside of wing): high pressure
- Flow always desires to go from high pressure to low pressure
- Flow 'wraps' around wing tips

- **Effect of Finite Wings:**
  - **Less Lift (than the equivalent airfoil)**
  - **More Drag (than the equivalent airfoil)**

Finite Wing Downwash

- Wing tip vortices induce a small downward component of air velocity near wing by dragging surrounding air with them.
- Downward component of velocity is called **downwash, w**



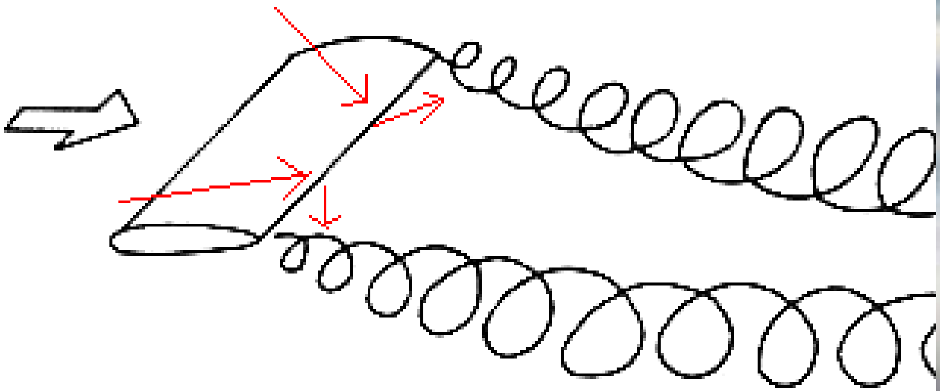
- **Two Consequences:**
  1. Increase in drag, called **induced drag (drag due to lift)**
  2. Angle of attack is effectively reduced, α_{eff} as compared with V_∞

Wing Tip Vortices

…cont'd

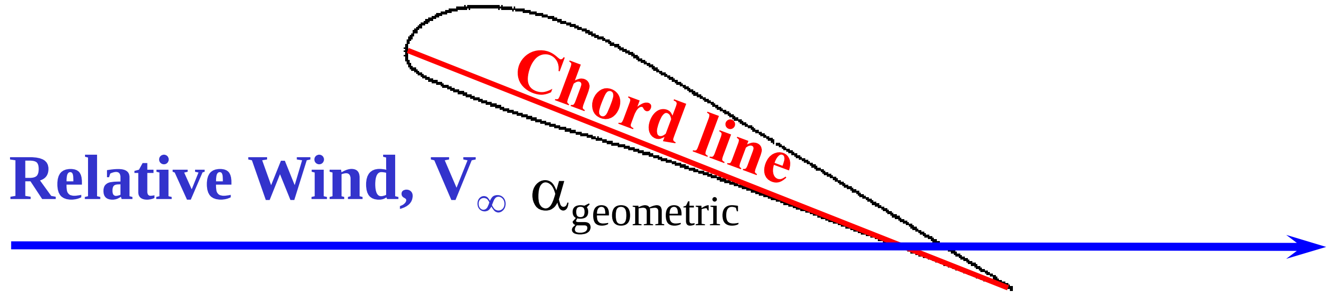
- Over the top of the wing the airflow goes inward
- Under the bottom of the wing the airflow moves outward

Due to the pressure changes

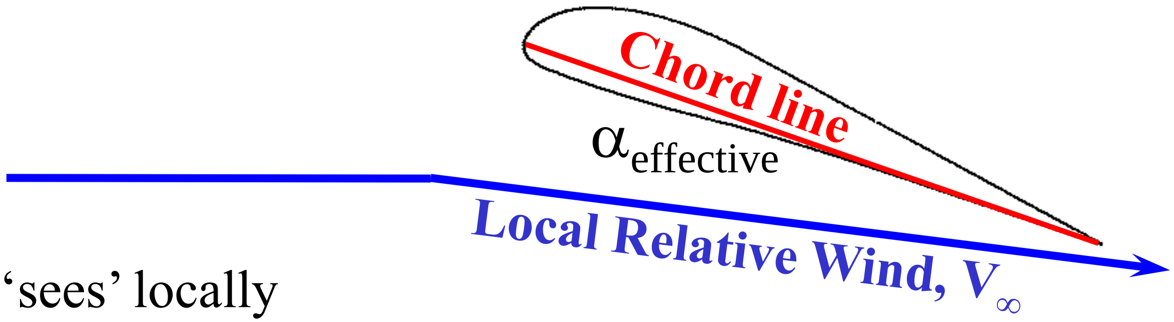


This creates a swirling motion off the wing tip called a vortex

Angle of Attack Definitions



- ❑ $\boldsymbol{\alpha}_{\textbf{geometric}}$: what you see, what you would see in a wind tunnel
  - ○ Simply look at angle between **incoming relative wind** and **chord line**
- ❑ This is a case of no wing-tips (infinite wing)



- $\boldsymbol{\alpha}_{\textbf{effective}}$: what the airfoil 'sees' locally
  Angle between **local flow direction** and **chord line**
  Small than $\alpha_{geometric}$ because of downwash
- The wing-tips have caused this local relative wind to be angled downward

ANGLE OF ATTACK DEFINITIONS

$$\alpha_{geometric} = \alpha_{effective} + \alpha_{induced}$$

$\boldsymbol{\alpha}_{\mathbf{geometric}}$: what you see, what you would see in a wind tunnel

Simply look at angle between **incoming relative wind** and **chord line**

$\boldsymbol{\alpha}_{\mathbf{effective}}$: what the airfoil 'sees' locally

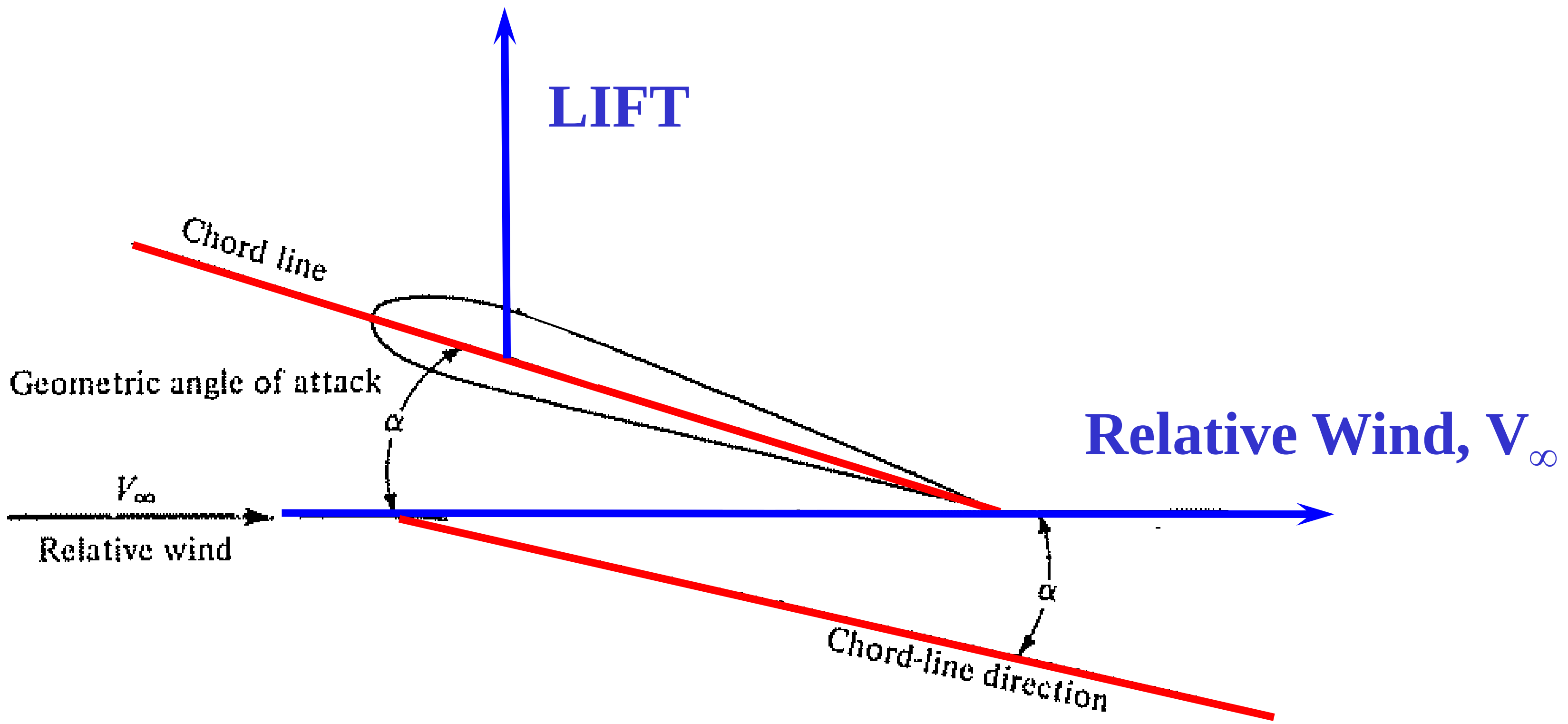
Angle between local flow direction and chord line

Small than $\alpha_{geometric}$ because of downwash

$\boldsymbol{\alpha}_{\mathbf{induced}}$: difference between these two angles

Downwash has 'induced' this change in angle of attack

INFINITE WING DESCRIPTION

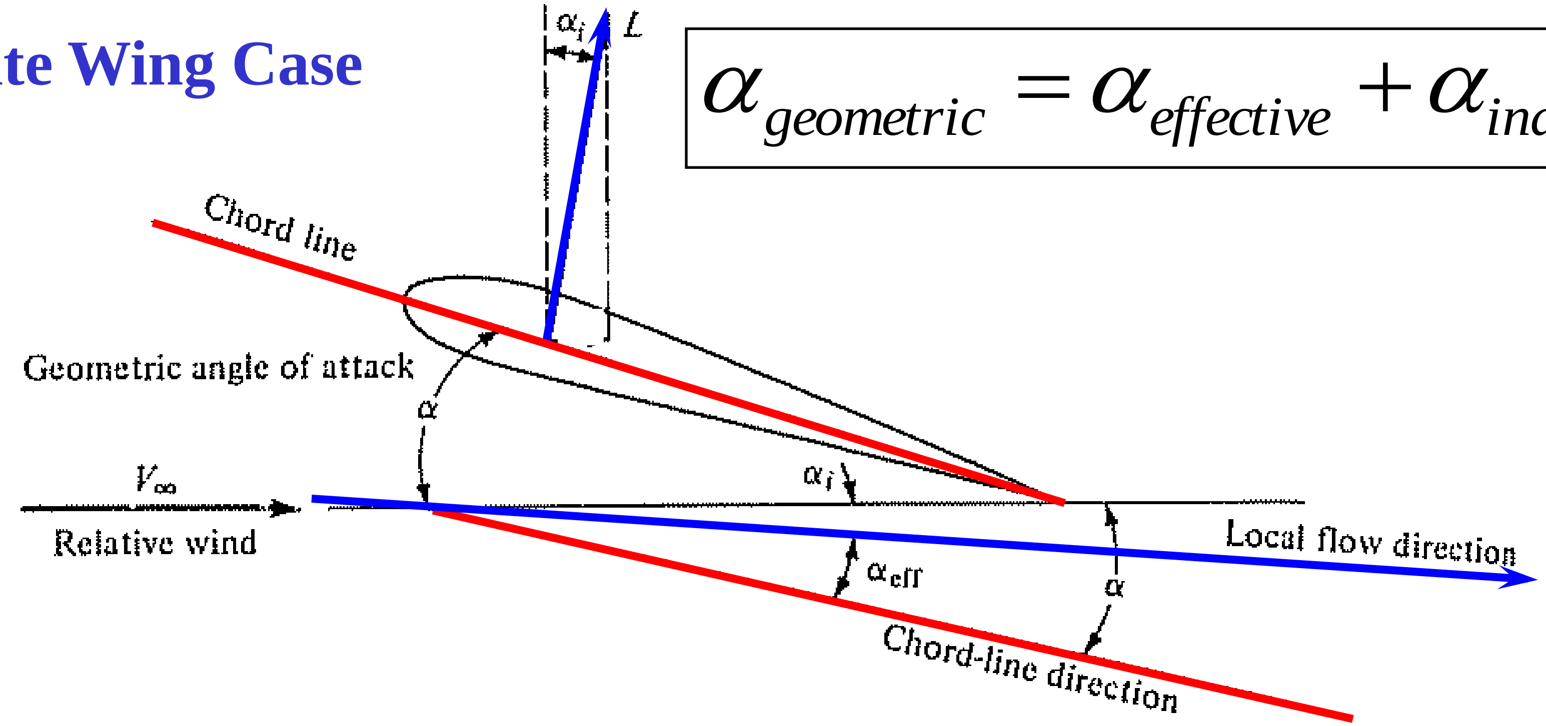


- LIFT is always perpendicular to the RELATIVE WIND
- All lift is balancing weight

FINITE WING DESCRIPTION

Finite Wing Case

$$\alpha_{geometric} = \alpha_{effective} + \alpha_{induced}$$

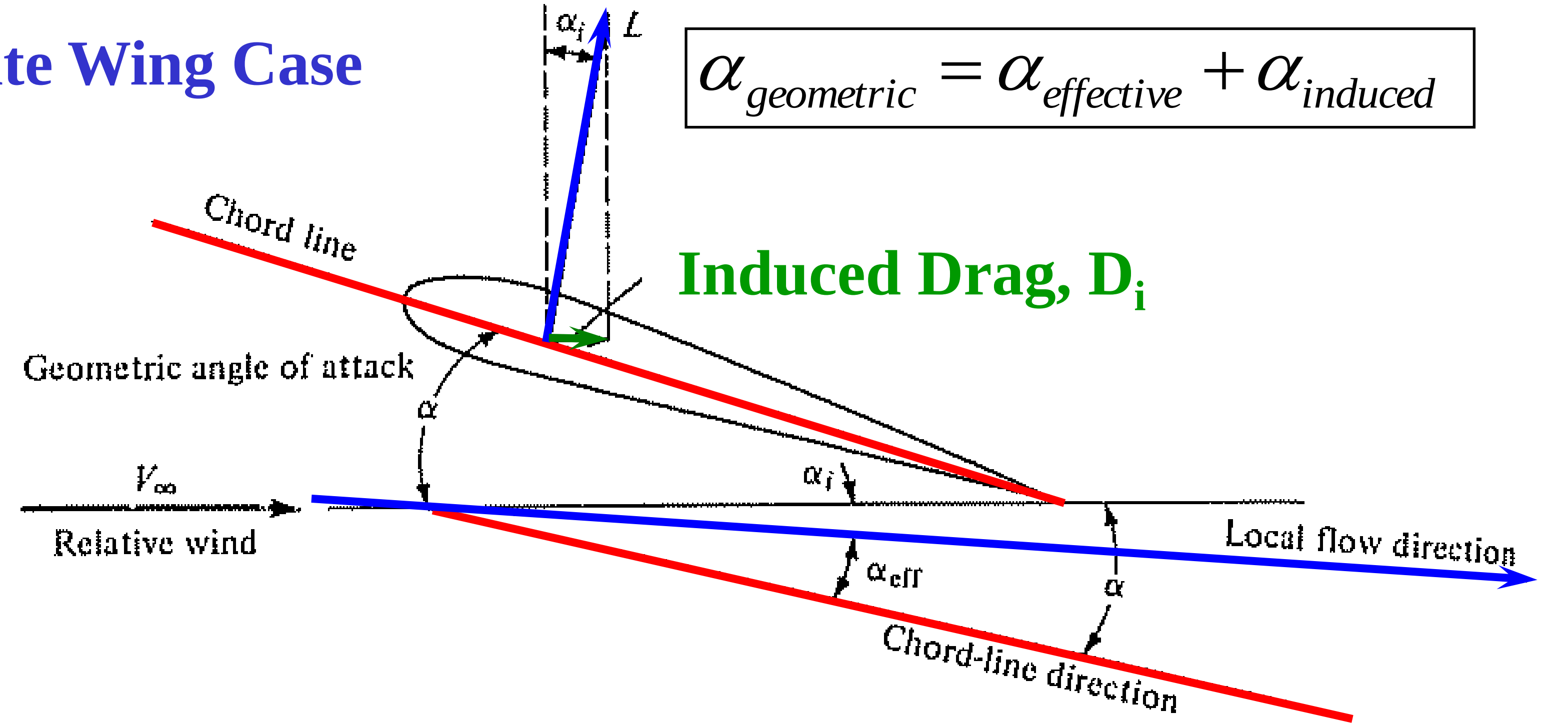


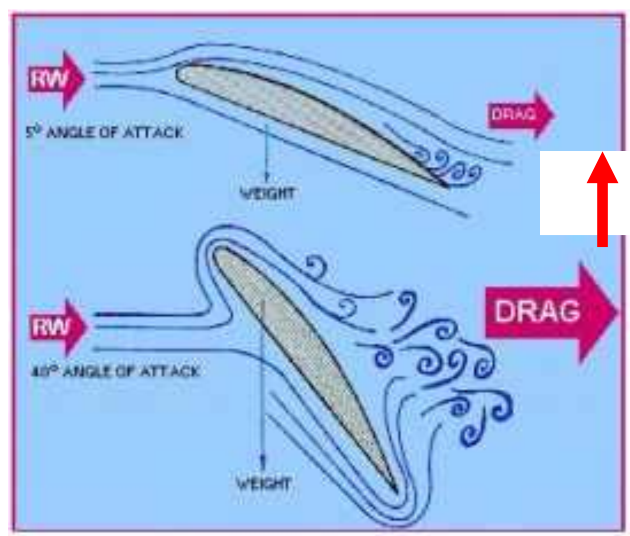
- Relative wind gets tilted downward under the airfoil
- LIFT is still always perpendicular to the RELATIVE WIND

FINITE WING DESCRIPTION

Finite Wing Case

$$\alpha_{geometric} = \alpha_{effective} + \alpha_{induced}$$



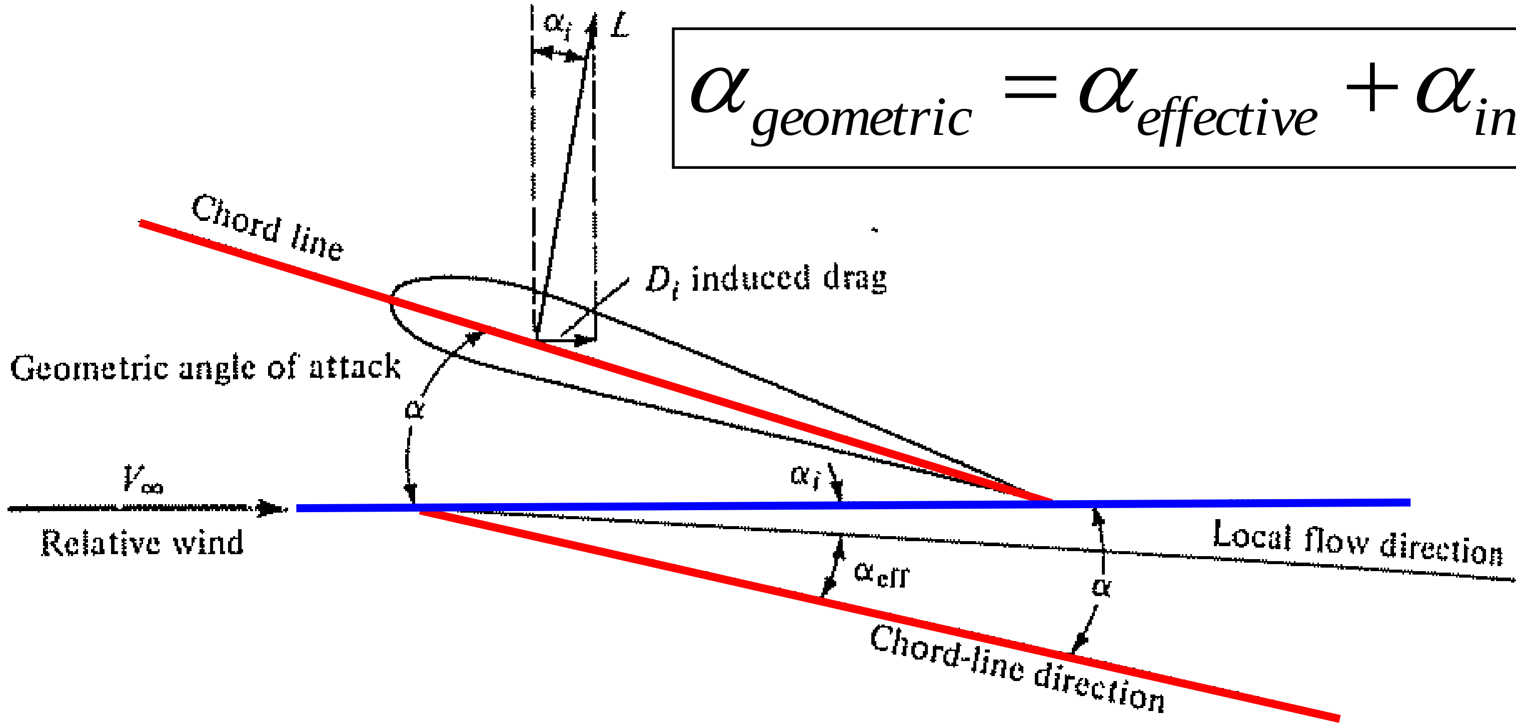


↑ **Lift** **Drag** ↑

- **Induced Drag :** Caused by those parts of an airplane which are active in producing lift
- Cannot be eliminated

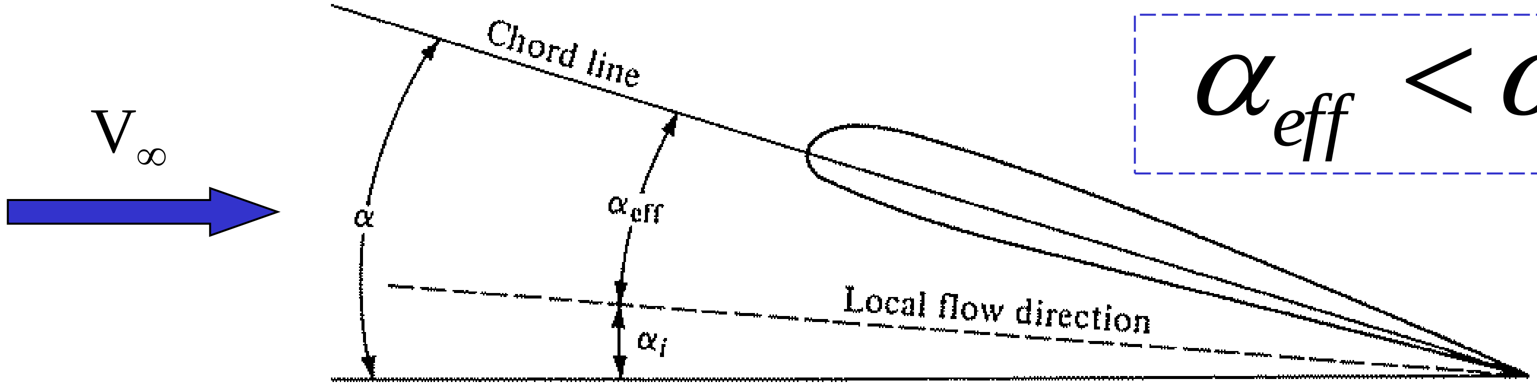
PHYSICAL INTERPRETATIONS

$$\alpha_{geometric} = \alpha_{effective} + \alpha_{induced}$$

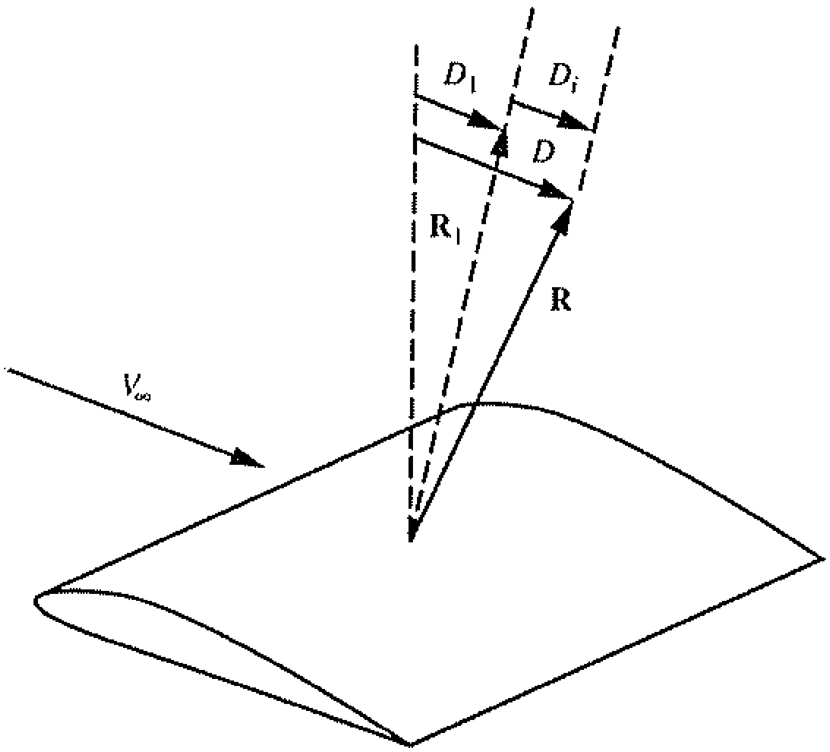


1. Local relative wind is canted downward, lift vector is tilted back so a component of L acts in direction normal to incoming relative wind
2. Wing tip vortices alter surface pressure distributions in direction of increased drag
3. Vortices contain rotational energy put into flow by propulsion system to overcome induced drag

INDUCED DRAG: IMPLICATIONS FOR WINGS



$$\alpha_{eff} < \alpha$$

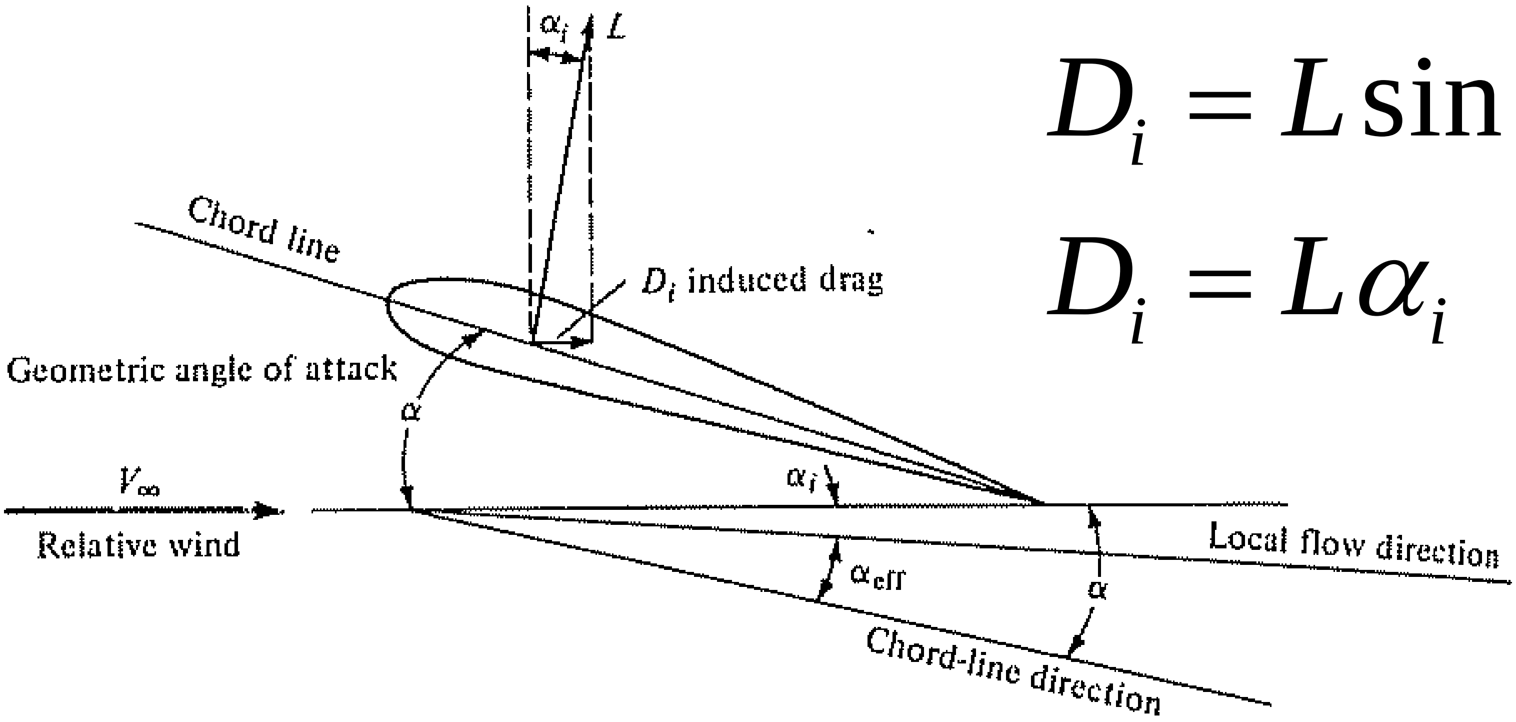


Finite Wing	Infinite Wing (Appendix D)

$$C_L < c_l$$

$$C_D > c_d$$

HOW TO ESTIMATE INDUCED DRAG



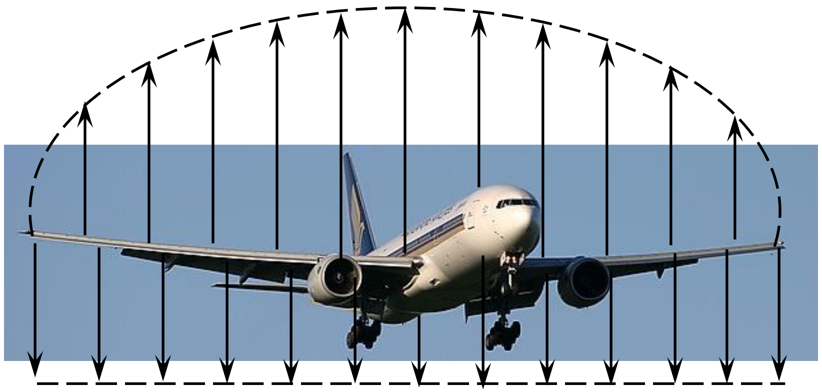
$$D_i = L \sin \alpha_i$$

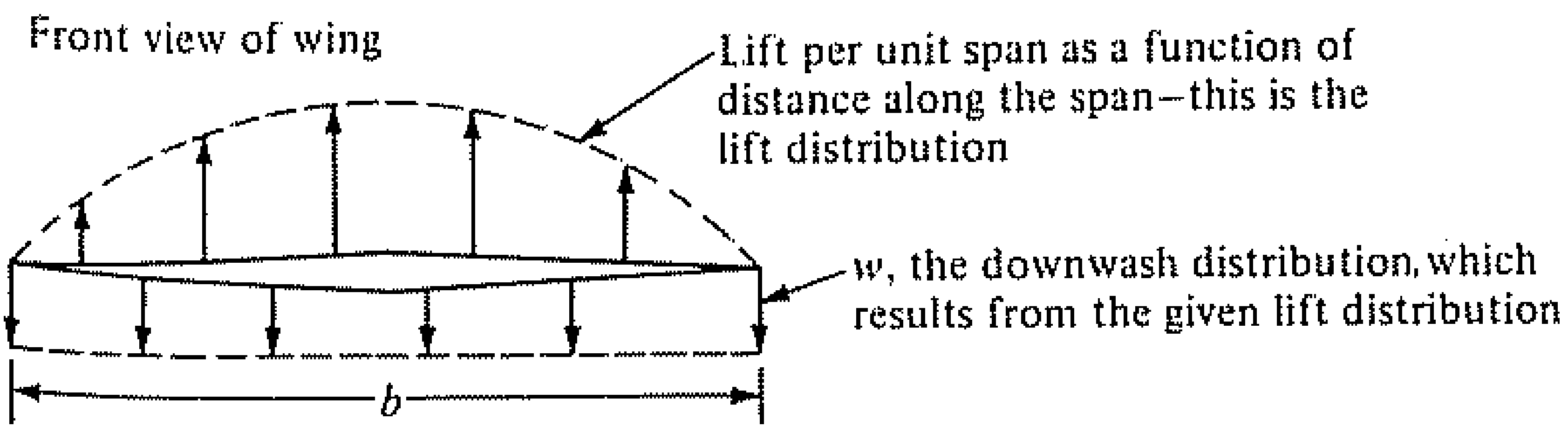
$$D_i = L \alpha_i$$

- Local flow velocity in vicinity of wing is inclined downward
- Lift vector remains perpendicular to local relative wind and is tiled back through an angle α_i
- Drag is still parallel to freestream
- Tilted lift vector contributes a drag component

HOW TO ESTIMATE INDUCED DRAG

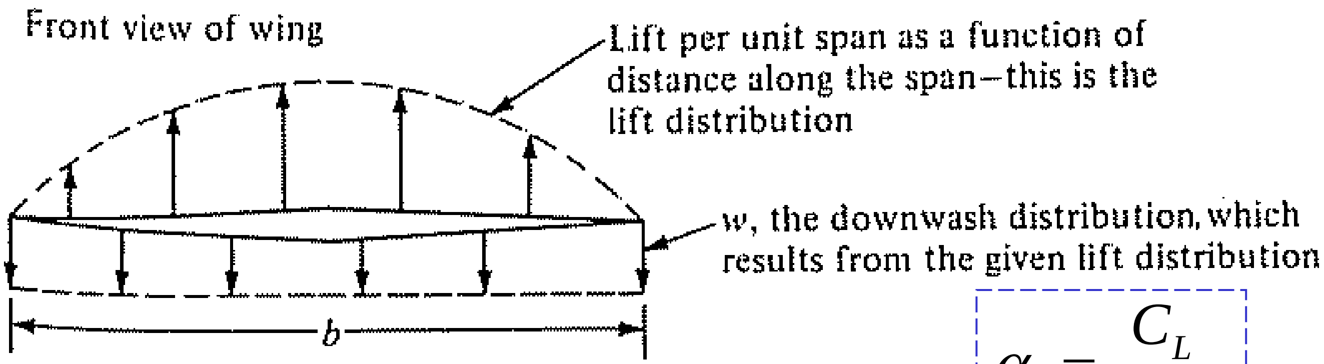
- Calculation of angle α_i is not trivial (MAE 3241)
- Value of α_i depends on *distribution of downwash* along span of wing
- Downwash is governed by *distribution of lift* over span of wing





HOW TO ESTIMATE INDUCED DRAG

- **Special Case: Elliptical Lift Distribution (produced by elliptical wing)**
- Lift/unit span varies elliptically along span
- This special case produces a uniform downwash



$$\alpha_i = \frac{C_L}{\pi AR}$$

$$D_i = L\alpha_i = L\frac{C_L}{\pi AR} = q_\infty S\frac{C_L^2}{\pi AR}$$

$$\frac{D_i}{q_\infty S} = \frac{C_L^2}{\pi AR}$$

$$C_{D,i} = \frac{C_L^2}{\pi AR}$$

Key Results:
Elliptical Lift Distribution

ELLIPTICAL LIFT DISTRIBUTION

- For a wing with same airfoil shape across span and no twist, an elliptical lift distribution is characteristic of an elliptical wing plan form
- Example: Supermarine Spitfire

Key Results:
Elliptical Lift Distribution

$$\alpha_i = \frac{C_L}{\pi AR}$$

$$C_{D,i} = \frac{C_L^2}{\pi AR}$$

HOW TO ESTIMATE INDUCED DRAG

- For all wings in general
- Define a span efficiency factor, e (also called span efficiency factor)
- Elliptical planforms, e = 1
  - The word planform means shape as view by looking down on the wing
- For all other planforms, e < 1
- 0.85 < e < 0.99

$$C_{D,i} = \frac{C_L^2}{\pi e AR}$$

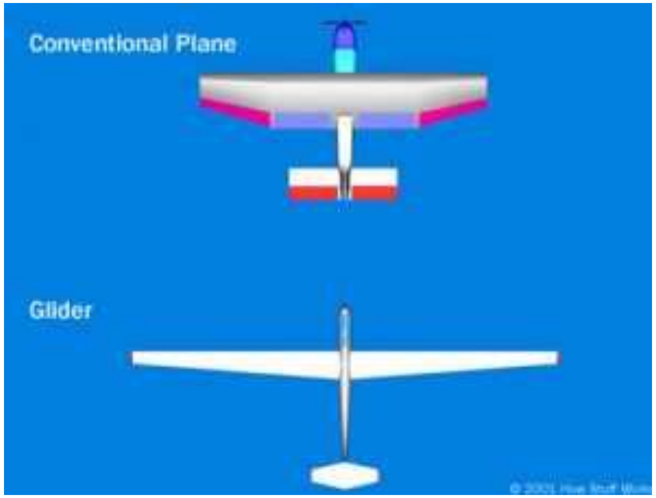
Goes with square of C_L
Inversely related to AR

Drag due to lift

Span Efficiency Factor

To reduce induced drag

- ❑ Induced drag can be reduced with a high aspect ratio
- ❑ Streamlining:

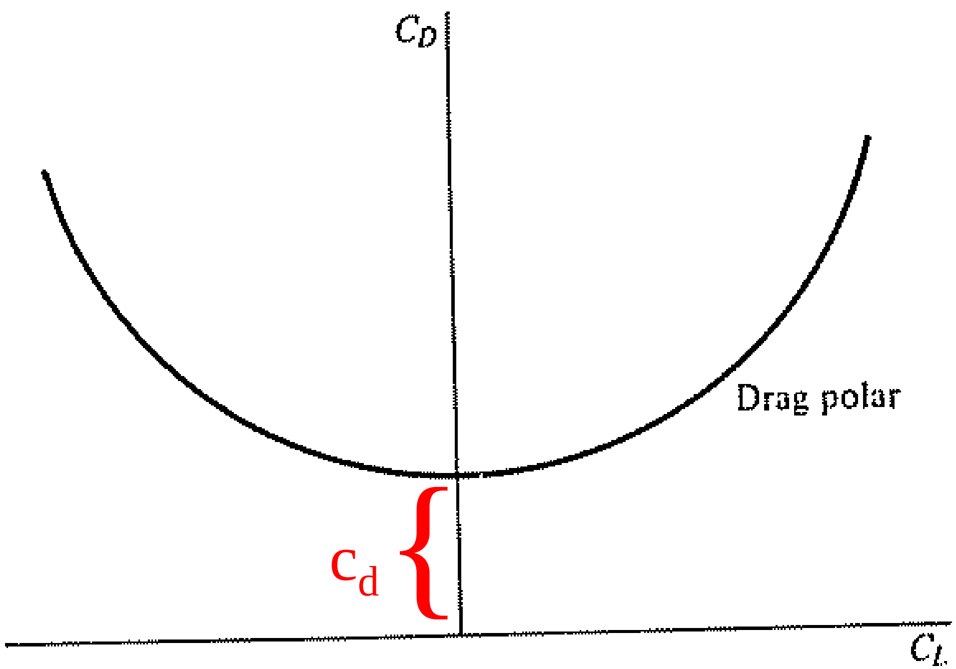


- The less drag you have…
  - Flying a glider: the further you can fly
  - Flying an airplane: the less fuel you use
- Therefore streamlining is important
  - A design device by which a body is shaped to minimize drag

DRAG POLAR EXAMPLE

$$C_D = c_d + \frac{C_L^2}{\pi e AR}$$

Total Drag = Profile Drag + Induced Drag



EXAMPLE: U2 VS. F-15

$$L = W = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$$

$$C_D = c_d + \frac{C_L^2}{\pi e AR}$$

- Cruise at 70,000 ft
  - Air density highly reduced
- Flies at slow speeds, low $q_\infty \rightarrow$ high angle of attack, high C_L
- U2 AR ~ 14.3 **(WHY?)**

- Flies at high speed (and lower altitudes), so high $q_\infty \rightarrow$ low angle of attack, low C_L
- F-15 AR ~ 3 **(WHY?)**

EXAMPLE: U2 SPYPLANE

$$C_D = c_d + \frac{C_L^2}{\pi e AR}$$

- Cruise at 70,000 ft
  - Out of USSR missile range
  - Air density, ρ_∞, highly reduced
- In steady-level flight, L = W

$$L = W = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$$

- As ρ_∞ reduced, C_L must increase (angle of attack must increase)
- AR ↑ C_D ↓
- U2 AR ~ 14.3

U2 stall speed at altitude is only ten knots (18 km/h) less than its maximum speed

EXAMPLE: F-15 EAGLE

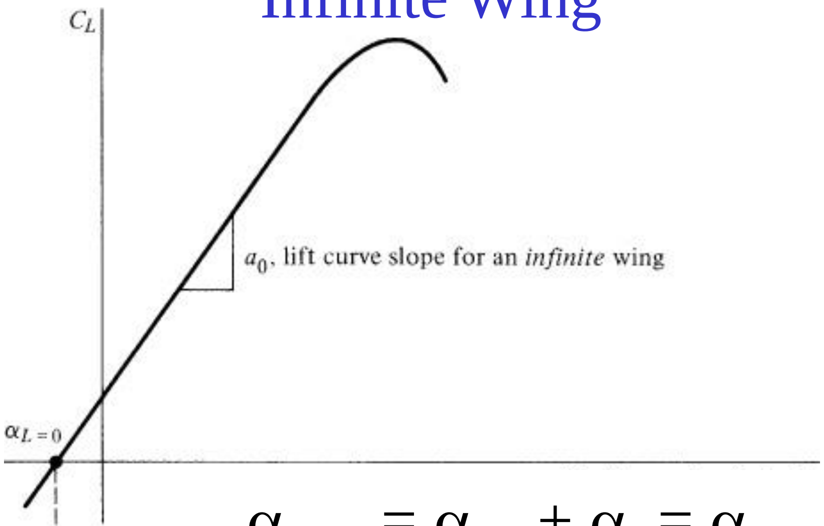
$$C_D = c_d + \frac{C_L^2}{\pi e AR}$$

- Flies at high speed at low angle of attack → low C_L
- Induced drag < Profile Drag
- Low AR, Low S

$$L = W = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$$

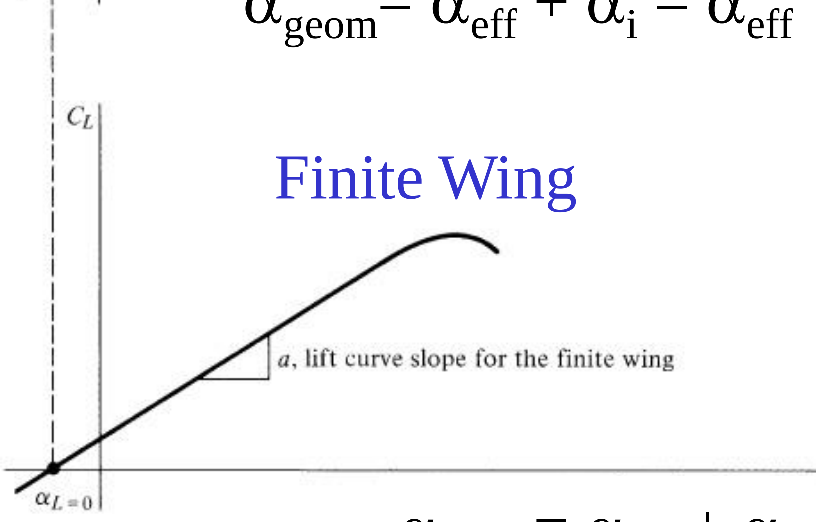
FINITE WING CHANGE IN LIFT SLOPE

Infinite Wing



$$\alpha_{geom} = \alpha_{eff} + \alpha_i = \alpha_{eff}$$

Finite Wing



$$\alpha_{geom} = \alpha_{eff} + \alpha_i$$

- In a wind tunnel, the easiest thing to measure is the geometric angle of attack
- For infinite wings, there is no induced angle of attack
  - The angle you see = the angle the infinite wing 'sees'
- With finite wings, there is an induced angle of attack
  - The angle you see ≠ the angle the finite wing 'sees'

$$\alpha_{geom} = \alpha_{eff} + \alpha_i$$

Summary: Infinite Vs. Finite Wings

- Properties of a finite wing differ in two major respects from infinite wings:
  1) Addition of induced drag
  2) Lift curve for a finite wing has smaller slope than corresponding lift curve for infinite wing with same airfoil cross section
- **Induced drag is price you pay for generation of lift**
- **$C_{D,i}$ proportional to C_L^2**
  - Airplane on take-off or landing, induced drag major component
  - Significant at cruise (15-25% of total drag)
- **$C_{D,i}$ inversely proportional to AR**
  - Desire high AR to reduce induced drag
  - Compromise between structures and aerodynamics
  - AR important tool as designer (more control than span efficiency, e)
- **For an elliptic lift distribution, chord must vary elliptically along span**
  - Wing planform is elliptical
  - Elliptical lift distribution gives good approximation for arbitrary finite wing through use of span efficiency factor, e

SUMMARY

- **Induced drag is price you pay for generation of lift**

- **$C_{D,i}$ proportional to C_L^2**
  - Airplane on take-off or landing, induced drag major component
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- **$C_{D,i}$ inversely proportional to AR**
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- **For an elliptic lift distribution, chord must vary elliptically along span**
  - Wing planform is elliptical
  - Elliptical lift distribution gives good approximation for arbitrary finite wing through use of span efficiency factor, e

End of Chapter

Thank you!